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The ongoing impact of modular localization on particle theory

To the memory of Hans-Jürgen Borchers (1926-2011)

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Abstract

Modular localization is the concise conceptual formulation of causal localization in the setting of local quantum physics. Unlike QM it does not refer to individual operators but rather to ensembles of observables which share the same localization region; as a result it explains the probabilistic aspects of QFT in terms of the impure KMS nature arising from the local restriction of the pure vacuum. Whereas it plays no important role in perturbation theory, as a result of representing the S-matrix as a relative modular invariant of wedge-localization it becomes indispensable for understanding analytic and algebraic properties of on-shell objects as the S-matrix and formfactors.

This leads not only to a new critical evaluation of the dual model and string theory, but also identifies ideas of embedding and dimensional reduction as inconsistent with the holistic properties of localization. Instead it reveals the conceptual origin of true particle crossing and points the way to a new formulation of Mandelstam's on-shell project of the 60s.

Modular localization also shows that perturbative calculations in the Krein-space setting can be better done directly in Hilbert space with the help of short-distance lowering string-localized potentials. This points to a vast extension of renormalizability for any spin.

Contents

1 Introduction

The course of quantum field theory (QFT) was to a large extend determined by three important conceptual conquests: its 1926 discovery by Pascual Jordan in the aftermath of what in recent times is often referred to as the *Einstein-Jordan conundrum* [1] [2] (a fascinating dispute between Einstein and Jordan), the discovery of renormalized perturbation in the context of quantum electrodynamics (QED) after world war II, and the non-perturbative insights into the particle-field relation initiated in the Lehmann-Symanzik-Zimmermann (LSZ) work on scattering theory which subsequently was derived from first principles [3] and applied to strong interactions in the context of the rigorous derivation of the particle analog of the Kramers-Kronig dispersion relations including their subsequent successful experimental test which extended the trust in QFT's foundational causality principle. These results encouraged a third project: particle-based on-shell formulations as the S-matrix bootstrap and Mandelstam's more analytic formulation in terms of auxiliary two-variable representations of elastic scattering amplitudes. The later gauge theory of the Standard Model resulted from an extension of the quantization ideas which already had led to QED. Besides many successes, it led to most of the still open problems of actual research.

Jordan's changed view about Einstein's statistical mechanics argument in favor of the existence of photons in his dispute with Einstein [2] not only led him to accept Einstein's reasoning, but also helped him to an extension of quantization to matter waves. But its main point, the thermal character of subvolume fluctuation resulting from the restriction of the global vacuum state to the observables localized in that subvolume did not receive the conceptual attention which, being a characteristic property which distinguishes QFT from quantum mechanics (QM), it would have deserved. In fact as a result of its incomplete understanding it became known as the "E-J conundrum" [1]. The perception of the stochastic thermal nature of the reduced vacuum state, i.e. the fact that the restriction of the pure global vacuum state to observables localized in a subregion behaves as an impure KMS state was not understood and this lack of understanding continued for many decades.

The by hindsight obvious explanation is that for a very long time QFT was not recognized as a theory following different principles from QM, but rather as some relativistic form of QM with infinite degrees of freedom. But in QM the vacuum does not become impure by spatial restriction, and this is independent of whether one uses the few degrees of freedom Schrödinger description or its infinite degree of freedom Fock space (second quantized) description. Many decades later when, in the special context of wedge-localization, this aspect of QFT was first noticed in form of an Gedankenexperiment[1] [4][5][6], its setting was too special and contrived in order to suspect the existence of an insufficiently understood additional foundational structure within causal quantum theory (QT). By the time of Unruh's observation, the E-J conundrum as a subject of further research was forgotten; it only played a role in historical reviews as [1] in which the opinion that the purity of the ground state and its tensor factorization in a spatial bipartite inside/outside separation is uphold.

¿From a modern point of view the d=1+1 Jordan model of a chiral current (in his view a "2-dim. photon field") is the simplest illustration of "localization-thermality" (LT) since it leads to a mathematical isomorphism [2] between LT and the global heat bath thermal behavior (HT) of statistical mechanics. In the Unruh Gedankenexperiment there is an *analogy* between LT and HT, but no isomorphism of the Unruh LT system to HT (i.e. no "inverse" Unruh effect in the sense of [7]).

Both Gedankenexperiments demonstrate a kind of thermal manifestation of causal localization whose early comprehension could have changed the path of QFT history. When Jordan's incomplete calculation was published as a separate section in the famous 1926 Dreimännerarbeit with Born and Heisenberg, his coauthors had some reservations, since it contained problematic aspects which had no place in the previously discovered QM; but they were not able to clearly articulate their doubts.

Several years later Heisenberg challenged Jordan in a letter about a missing logarithmic term proportional to $ln\varepsilon$ in his calculation of the fluctuation spectrum (where ε is a length which characterizes the "fuzzyness" deviation from sharp localization) at the endpoints of Jordan's localization interval [1]. This led to Heisenberg's discovery of *vacuum fluctuation* near the localization boundaries with ε the "attenuation length" ("fuzzyness" of localization-boundary) conceeded to the vacuum polarization cloud. As we know nowadays, the localization-caused vacuum polarization (VP) and LT are opposite sides of the same coin. Jordan's missed logarithmically divergent localization entropy resulting in the sharp localization limit (with the "roughness ε" at the endpoints) for $\varepsilon \to 0$ is the one-dimensional counterpart of the dimensionless area law A/ε^2 for *localization entropy*

[1] For the present purpose it does not matter whether the reader thinks that the Unruh temperature can be measured by a thermometer (incorrect!) or whether it is just a parameter which describes the KMS impurity of a restricted vacuum state (correct) which is a consequence of modular localization [6].

with $\varepsilon =$ "roughness" (attenuation length conceded to the VP cloud[2]) [9] [10].

These somewhat hidden properties of QFT place this theory into a sharp conceptual contrast to QM; it is neither QM with infinite degrees of freedom nor should it be referred to as "relativistic QM" [10]. In his famous paper on VP which he wrote after challenging Jordan about the missing $ln\varepsilon$ contribution, Heisenberg showed that the localization of dimensionless quantum charges ("partial charge") in QFT behaves quite different from their counterpart in QM. The inverse relation between sharpness of localization boundaries and increase of VP, measured in terms of the amount of entropy, is the QFT *substitute of the uncertainty relation* (which as the absence of the position operator among localized observables has no place in QFT). Relativistic QM built on the cluster factorization property, which deals directly with particles without the mediation of fields ("direct particle interactions"), does exist, but besides a Poincaré-invariant S-matrix (no crossing property) it has no local covariant observables which characterize causal QFT [11].

The algebraic formulation of QFT, often referred to as *local quantum physics* (LQP) or algebraic QFT (AQFT), has brought the localization properties into the forefront by demonstrating that they have a natural *mathematical counterpart*: *the Tomita-Takesaki modular theory of operator algebras* [3]. The more recent terminology "modular localization" refers to its deep connection to the causal localization principle, which identifies QFT as *that* quantum theory (QT) which results from the mathematical implementation of this principle. It refers directly to localized subspaces and subalgebras instead of individual states and operators; the role of quantum fields is simply to "coordinatize" localized algebras by playing the role of pointlike *singular generators of all localized algebras.* Unlike QM, it does not refer to events associated with *individual* observables, but rather deals with *ensembles* of observables (idealized as localized algebras) which share the same spacetime localization region. As a consequence QFT leads via LT and the resulting KMS property to a statistical notion, the same real probability as in statistical mechanics.

This is quite interesting from a conceptual point of view since Born's probability postulate in QM has been a point of philosophical controversies. The realization that probability is an unavoidable consequence of the quantum realization of Einstein's Minkowski space formulation of the Faraday/Maxwell causality (action at the neighborhood principle) would probability have pleased Einstein, who had a lifelong problem with Born's assignment of probability to individual events (or to "imagined" ensembles) in order to be able to interpret the Heisenberg/Schrödinger QM. In section 3 and 4 some of its important definitions and consequences of modular localization will be presented.

The foundational role of modular localization begs the question: how was it possible to set up renormalized perturbation theory (the textbook QFT of Lagrangians quantization) without a thorough understanding of the foundational role of the causal localization principle and its consequences?

The answer is surprisingly simple: in overcoming the old (Heitler, Wentzel) QM- in-

[2]The only infinities in QFT occur in quantities which depend on the VP cloud in the limit of sharp localized boundaries. The ultraviolet divergence problems in connection with calculations of renormalized correlation functions are the result of using ideas from QM instead of treating quantum fields as operator-valued Schwartz distributions [12].

spired formulation, it was sufficient for Tomonaga, Feynman, Schwinger and Dyson to invoke covariance, which is related to localization, but not equivalent to it. The remaining problems took the form of consistent prescription of how to handle infinities in terms of cutoffs or regulators. Even later, after Epstein and Glaser [12] showed that an inductive use of causal locality, which combined with a minimality requirement on the short-distance scaling limit leads to the renormalized result in a completely finite way, the above mentioned subtleties of causal localization still did not play a role. In retrospect one may say that modular localization entered particle physics for the first time through Sewell's observation [5] that the identification of the Bisognano-Wichmann identification modular group of wedge-localized algebras with the wedge-preserving Lorentz boost and the reparametrization of the boost parameter in terms of the proper time of an accelerated observer explains the Unruh effect as a special LT aspect of modular localization. But it took another three decades to unravel its constructive power.

Although modular localization had no direct impact on renormalized perturbation, it would be premature to conclude that its structural consequences are limited to E-J, the Unruh Gedankenexperiment and Hawking radiation. Recent conceptual progress in QFT revealed that LT explains the conceptual origin of the particle crossing property in on-shell quantities as the S-matrix and formfactors [10].

The particle crossing property was still unknown when Heisenberg attempted to formulate particle theory directly in terms of the S_{scat} matrix [13] without referring to fields, whose insufficiently understood inherent singular character (Laurent Schwartz distributions) led to the "ultraviolet catastrophe". With the derivation of the LSZ scattering theory and certain analytic properties (needed in the derivation of the particle counterpart of the Kramers-Kronig optical dispersion relations) also the *crossing property* received attention. Through its perturbative identification in mass-shell restricted Feynman graphs, it became gradually clear that particle theory contained a mysterious analytic on-shell property, in which incoming particles became interchanged with analytically continued momenta outgoing anti-particles. It was not possible to reduce this property to the (by that time already known) analytic properties of Wightman's [27] off-shell correlation functions (the Bargman-Wightman-Hall analytic domain). A rigorous derivation for special elastic scattering amplitudes from locality properties of off-shell 4-point functions was based on the use of the unwieldy mathematical theory of several complex variables [14]; as a result the conceptual origin of particle crossing remained a mysterious issue within the anyhow poorly understood field-particle relation beyond LSZ scattering theory.

These incomplete attempts to unravel the nature of analytic particle crossing were mostly ignored in mainstream particle physics; they did not fit the post QED Zeitgeist of S-matrix research in order to understand strong interactions, which mainly consisted in inventing computational rules and adding analytic assumptions as the computations progressed. In retrospect it is clear that a foundational understanding of on-shell analytic properties from the causality principle of QFT was beyond the conceptual knowledge at that time. As Heisenberg's first S-matrix attempt, also the bootstrap project came soon to a halt for the same reasons: the underlying principles were too general and the additional analytic working assumptions too vague and ad hoc in order to serve for the start of meaningful computations. Their nonlinear nature (unitarity "by hand" and not through the large time asymptotic scattering limit of linear field operators) created the wrong expectation that if the bootstrap admits a solution at all, it should be rather unique (a precursor of later "theories of everything"). Stanley Mandelstam [15], one of the most dedicated champions of a "top-to-bottom approach"[3] based on observable on-shell objects, tried to make the S-matrix project more amenable to calculations by adding reasonably-looking assumptions concerning two-variable spectral representation for the elastic scattering amplitude; in fact he introduced most of the on-shell terminology whose use became standard and will certainly most other ideas of that epoch.

It is the main aim of the present work to show that this project took a wrong turn away from its original purpose of an S-matrix-based on-shell construction in particle theory, when in the late 60s a crossing property based on mathematical properties of Euler's beta functions was proposed [16]. This led to the dual model and finally to string-theory. The defining function of the dual model is a crossing symmetric meromorphic function whose analytic crossing property turns out to have no relation to particle crossing. Actually the particle crossing in the S-matrix and formfactors can for structural physical reasons not be described by a meromorphic function in the Mandelstam variables; not even an approximand which violates unitarity but maintains the other properties of scattering amplitudes can be meromorphic (absence of cuts) in s,t,u. This raises the question whether the meromorphic dual model function can be related with *any* property in quantum physics or whether it remains the solution of an entirely mathematical game as its origin suggests.

Following observations by Gerhard Mack [17][18], it will be shown in the next section that the meromorphic dual model crossing is a *rigorous property* of the Mellin transform of *conformal* 4-point-functions. The location of the poles of this function is given by the anomalous dimensional spectrum which has no bearing on particle physics. In fact not only Veneziano's dual model but also all later versions are of this form. This somewhat unexpected property is a reflection of what in the next section is called the "picture puzzle" appearance of the analogy between (d, s) scale dimension spectra in conformal QFTs and the (m^2, s) particle spectra in QFT with mass gaps. Similar to certain generalized free fields [28], models with discrete infinite particle spectra violate the causal completeness property and cause the appearance of a Hagedorn temperature in thermal states whereas infinite discrete spectra of conformal scale dimensions do not show these unphysical properties. Therefore it is important to be more explicit about the mathematical properties of this "picture puzzle" scale dimension/mass relation. The precise statement about the particles from ST and the dual model is of an entirely group theoretic nature and establishes the *existence of a positive energy Poincaré representation on the irreducible oscillator algebra of a particular 10 dimensional chiral conformal current model*. This group theoretic fact may be surprising, but it bears no relation to interacting particles and their S-matrix.

In section 4 we return to the problem of the true origin of particle crossing in the S-matrix and formfactors and its use in on-shell constructions; it will be shown that the particle crossing identity is nothing else as *the KMS property* associated with wedge localization and rewritten in terms of *emulated free-field associated particle states (*a new concept from modular localization). It is very pleasing that the recognition of the failure of

[3] An alternative of the standard "bottom-to-top quantization" with its intermediate ultraviolet problems in which the physical interpretation (top) starts after the end of the computations.

the old S-matrix approach is also the start of a new S-matrix-based on-shell construction (section 4), this time based on modular localization.

As a preparatory step for section 4 one needs to know some basics about modular operator theory. This is the purpose of *section 3* which starts by explaining the limitations of the standard way of covariantizing Wigner's positive energy representation and how modular localization of wave functions helps to overcome them. The modular localization of subspaces prepares the ground for the modular localization of operator algebras which in turn leads to the Tomita-Takesaki modular theory in the LQP setting of operator algebras [3].

Historically the idea of modular localization of wave functions entered Wigner's representation theory as the result of trying to understand the resistance of the zero mass infinite spin class (faithful representation of Wigner's "little group") against any attempt to extract a covariant field from those representations. This problem was only solved more than 6 decades after Wigner's pathbreaking work with the help of modular localization [19][20] by realizing that this class of representations only admits semiinfinite string-localized, instead of pointlike generating wave function. This explained immediately why there was no classical analogue i.e. no Lagrangian from to which this wave function was related through a Euler-Lagrange variation[4]. Allowing string-like solutions also turned out to resolve the well-known clash of massless pointlike vectorpotentials with the Hilbert space positivity. The better alternative between the two possibilities (either pointlike in Krein space or stringlike in Hilbert space) is the latter. The same remedy applies to higher spin massless representation.

In section 3 it is also shown how this observation leads to a radical changes of the concept of renormalizability with a new view about remaining foundational problems of the Standard Model.

The concluding remarks tries to explain the origin of the deep schism between particle physics carried out in the critical tradition (as in sections 2, 3 and 4 in the present work) and the more metaphoric ST-influenced majority view of what constitutes particle theory.

Our findings support the title and the content of an important contribution by the late Hans-Jürgen Borchers in the millennium edition of Journal of Mathematical Physics [21] which reads : "Revolutionizing Quantum Field Theory with Tomita-Takesaki's modular theory". With all reservations about misuses of the word "revolution" in particle physics, this paper is a comprehensive account of the role of modular operator theory in LQP; its title may also be seen as a premonition of the present progress which is driven by concepts coming from modular localization. LQP ows Borchers many of the ideas coming from modular operator theory; for this reason it is very appropriate to dedicate the present article to his memory.

The new insight which permits to view this new setting among other things also as a legitimate heir of Mandelstam's S-matrix ideas before ST, is the observation that the S-matrix, in addition to its well known role in scattering theories, is also a *relative modular invariant between the wedge-localized interacting algebra and its free counterpart* (generated by the incoming free fields smeared with wedge-supported test functions). This new role of the S-matrix was already implicitly contained in Res Jost's work on

[4]As a rule of thumb (consistent with all that is known): string-localized fields are not Euler-Lagrange and Euler-Lagrange objects (ST a la Polyakov) are not string-localized.

the TCP theorem in the setting of a complete particle interpretation; but it only found its first constructive application after it was realized that the Zamolodchikov-Faddeev algebra operators of integrable d=1+1 models *are the generators of spacetime-localized wedge algebras* in the setting of integrable QFTs [22][23].

Some historical remarks may facilitate to understand the motivation behind this paper; which after all addresses foundational problems of QFT which developed over many decades. The natural conceptual framework in which the modular localization attained its important role is the algebraic LQP setting of QFT. It started with Haag's 1957 attempt[5] [25] to base QFT on *intrinsic principles* instead of subordinating a more fundamental theory via a *quantization parallelism* to a less fundamental classical field theory. The idea that a foundational theory as QFT should not be forced to "dance to the tune" (Jordan used the expression "classical crutches") coming from a less fundamental classical theory can already be found in some of Jordan's early work [26], but the necessary algebraic concepts were not yet available at his time. Hence the terminology LQP in the present work stands for a *different formulation* of QFT *while keeping its physical content*[6]. Another setting, which also did not refer to quantization, was Wightman's [27] formulation of quantum fields in terms of operator-valued Schwartz distributions and their correlation functions. The two approaches are conceptually closely related by viewing the Wightman fields as generators of local algebras.

The quantum aspects of causal localization and the associated maximal propagation speed have been the cause of innumerous misunderstanding. Even one of the most reputable research journals published an article in which Fermi's famous Gedankenexperiment to demonstrate that the classical limitation through the velocity if light passes to QED was thrown into doubt [31]. Following critical remarks, PRL published the correct arguments [32]. The "effective" localization of propagating (spreading) wave packets in QM (e.g. the velocity of sound) is not changed in QFT apart from the fact that there is a maximal effective velocity. But different from QM, the localized observables of QFT retain the exact (in contrast to effective) classical relativistic propagation properties in the foundational modular localization of LQP, although they now lead to completely different nonclassical consequences; fields as singular objects (operator-valued distributions), VP near localization boundaries ($\rightarrow$ localization entropy) and KMS properties of spacially restricted vacuum states including the natural appearance of probability without Born's help. More important is the algebraic aspect of causal localization which is characteristic of QFT. The rather comprehensive correct account in PRL did however not stop the appearance of "superluminal" papers whose error can always be traced back to the same misunderstanding of causal localization.

The presentation of results is strictly limited to their mathematical-conceptual content; only in the concluding section we allow ourselves some remarks about their position in the sociological-ideological struggle of the search of the "heart and soul" of particle theory. The origin of the present schism in particle theory is presented with the certain sadness of somebody who is able to compare the present situation with that when he entered particle theory at the beginning of the 60s. In the present situation in particle theory

[5]The original written version is in French, later it was translated back into English in [24]..

[6]The only reasson for using occasionally LQP instead of QFT is to remind the reader that the terminology QFT in this article represents more than a collection of calculational recipes.

the stagnation of real progress is hidden under an ever increasing mountain of uncritical speculative papers which instead of solving existing problems only produce new unsolved ones.

Owing to the subleties of the problems discussed in this paper repetitions with slight changes of emphasis are meant to be helpful for the reader.

2 Anomalous conformal dimensions, particle spectra and crossing properties

A large part of the conceptual derailment caused by string theory can be understood without invoking the subtleties of modular localization. This will be the subject of the following four subsections.

The principle of *modular localization* becomes however essential for a foundational understanding of the particle crossing property which is important for a new formulation of a constructive on-shell project based on the correct crossing property which replaces Mandelstam's attempt and is compatible with the principles of Haag's local quantum physics. This will be the subject of section 3 and 4.

2.1 Born localization versus causal localization

Since part of the misunderstandings in connection with ST have to some extend their origin in confusing "Born localization" in QM with the causal localization in QFT, it may be helpful to review their differences [11].

It is well-known since Wigner's 1939 description of relativistic particles [3] in terms of irreducible positive energy representations of the Poincaré group that *there are no covariant position operators* x_{op}^{μ}; in fact the impossibility to describe relativistic particles in terms of quantizing a classical relativistic particle action or in any other way was Wigner's main reason for his representation theoretical construction of their wave function spaces. The rather simple argument against covariant selfadjoint x_{op}^{μ} follows from the non-existence of covariant spectral projectors E

$$\begin{aligned} &\vec{x}_{op} = \int \vec{x} dE_{\vec{x}},\ R \subset \mathbb{R}^3 \to E(R) \qquad (1)\\ &U(a)E(R)U(a)^{-1} = E(R+a),\ E(R)E(R') = 0\ for\ R \times R' \\ &(E(R)\psi, U(a)E(R)\psi) = (\psi, E(R)E(R+a)U(a)\psi) = 0 \end{aligned}$$

where the second line expresses translational covariance and orthogonality of projections for spacelike separated regions. In the third line we assumed that the translation a shifted $E(R)$ spacelike to itself. But since $U(a)\psi$ is analytic in $\mathbb{R}^4 + iV^+$ (V^+ forward light cone) as a result of the spectrum condition, $\|E(R)\psi\|^2 = 0$ *for all* R and ψ which implies $E(R) \equiv 0$ i.e. covariant position operators do not exist.

The "Born probability" of QM results from Born's proposal to interpret the absolute square $|\psi(\vec{x},t)|^2$ of the spectral decomposition $\psi(\vec{x},t)$ of state vectors with respect to the spectral resolution of the position operator $\vec{x}_{op}(t)$ at time t. Its use as a localization probability density to find an individual particle in a pure state at a prescribed position became the beginning of one of a still not closed philosophical disputes in QM which Einstein entered through his famous saying: "God does not play dice".

In Haag's LQP setting this problem does not exist since, as previously mentioned, its objects of interests are not global position operators in individual quantum mechanical systems, but rather ensembles of causally localized operators which share the same spacetime localization i.e. belong to the spacetime-indexed algebras $\mathcal{A}(\mathcal{O})$ of Haag's LQP (next section). The modular localization attributes statistical mechanics-like KMS properties resulting from a highly impure *reduced* vacuum state to such an ensemble. As mentioned this leads to a completely intrinsic notion of probability. As in statistical mechanics, the KMS property *is inherited by all the individual operators* of $\mathcal{A}(\mathcal{O})$ there is no reason to use the Born's quantum mechanical probability interpretation[7].

Traditionally the causal localization of QFT enters the theory with the (graded) spacelike commutation (Einstein causality) in Minkowski spacetime of pointlike localized covariant fields. There are very good reasons to pass to another slightly more general, but in a subtle sense also more specific LQP formulation of QFT, namely to Haag's local quantum physics (LQP) in which the fields play the auxiliary role of (necessary singular) generators of local algebras[8]. In analogy to coordinates in geometry there are infinitely many such generators which generate the same algebra as there are different coordinates which describe the same geometry. As in Minkowski spacetime geometry these "field coordinates" can be chosen globally i.e. the same generating field for the generation of all local algebras associated to one LQP.

In this more conceptual LQP setting it is easier to express the *full* content of causal localization in a precise operational setting. It includes not only the Einstein causality for spacelike separated local observables, but also a timelike aspect of causal localization, namely the equality of an $\mathcal{O}$-localized operator algebra $\mathcal{A}(\mathcal{O})$ with that of its *causal completion* $\mathcal{O}''$

$$\begin{aligned} &\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''),\ \mathcal{O}' = causal\ disjoint\ of\ \mathcal{O},\ causal\ completeness \\ &\mathcal{A}(\mathcal{O}') \subseteq \mathcal{A}(\mathcal{O})',\ \ = Haag\ duality,\ \ \subset\ Einstein\ causality \end{aligned} \tag{2}$$

(with $\mathcal{A}(\mathcal{O})'$ commutant of $\mathcal{A}(\mathcal{O})$).The causal completeness requirement does not follow from Einstein causality and corresponds to the classical causal propagation. A closely related property is Haag duality. The advantage of the LQP formulation over the use of fields is clearly seen in case of these three properties for which only Einstein causality permits a simple formulation in terms of generating fields.

It is not evident but nevertheless true that this timelike causal completion aspect of causality is intimately related to the cardinality of phase space degrees of freedom. Whereas both properties are *formal* attributes of Lagrangian quantization, they have to be added in "axiomatic" settings based on mathematically controlled (and hence neither Lagrangian nor functional) formulations [28]. Their violations for subalgebras $\mathcal{A}(\mathcal{O})$ as a

[7]As mentioned there exists still the hope to derive Born:'s probability in QM as a relic of the intrinsic LQP probability in a conceptually better understood future relation of QFT with QM.

[8]To be more precise they are operator-valued Schwartz distributions whose smearing with $\mathcal{O}$-supported test functions are (generally unbounded) operators affiliated with a weakly closed operator algebra $\mathcal{A}(\mathcal{O})$.

result of too many phase space degrees[9] of freedom leads to physically undesirable effects, which among other things limit the physical application of the mathematical AdS-CFT correspondence (last subsection).

On the other hand the violation of Haag duality for disconnected or multiply connected regions have interesting physical consequences in connection with superselection sectors associated with observable algebras, or with the QFT Aharonov-Bohm effect for doubly connected spacetime algebras which has its simplest formulation in (m=0,s=1) Wigner representations with possible generalizations to multiply connected spacetime regions in higher spin (m=0,s>1) representations [33][34].

The LQP formulation of QFT is naturally related to the Tomita-Takesaki modular theory of operator algebras; its general validity for spacetime localized algebras of the latter is a direct result of the Reeh-Schlieder property [3] for localized algebras $\mathcal{A}(\mathcal{O}), \mathcal{O}'' \subset \mathbb{R}^4$ (next section).

It is important to understand that quantum mechanical localization is not cogently related with spacetime. A linear chain of oscillators simply does not care about the dimension of space in which it is pictured; in fact it does not even care if it is related to spacetime at all, or whether it refers to some internal space to which spacetime causality concepts are not applicable. The modular localization on the other hand is *imprinted* on causally local quantum matter, it is a totally *holistic* property of such matter. As life cannot be explained in terms of the chemical composition of a living body, localization does not follow from the mathematical description of the global oscillators (annihilation/creation operators) in a global algebra. Oscillator variables $a(p), a^*(p)$ "do not know" whether they will be used to define Schrödinger fields or free covariant local quantum fields. It is the holistic *modular localization principle* which imprints the causal properties of Minkowski spacetime (including the spacetime dimension) on operator algebras and thus determines in which way the irreducible system of oscillators will be used in the process of localization [35]; in QFT there is no abstract quantum matter as there is in QM; rather localization becomes an inseparable part of it. Contrary to a popular belief (the credo about dimensional reduction and extra dimensions), this holistic aspect of QFT (in contrast to classical theory and Born's localization in QM) *does not permit an embedding of a lower dimensional theory into a higher dimensional* one, neither is its inversion (Kaluza-Klein reduction) possible

One problem in reading articles or books on ST is that it is sometimes difficult to decide which localization they have in mind. When e.g. Polchinski in [36] uses the relativistic particle action $\sqrt{ds^2}$as a trailer for the introduction of the Nambu-Goto minimal surface action $\sqrt{d\sigma^2}$ (with $d\sigma^2$ being the quadratic surface analog of the line element ds^2) in a description of ST, he probably was unaware that this trailer is contraproductive because it suggests that a covariant quantum string may not exist for the same reasons as there are no covariant quantum particle operators. So what was intended as a trailer turns out to be more like a "squib load".

The Polyakov action is the square of the N-G action; it can be formally written in terms of the potential of an n-component chiral current

[9] For the notion of phase space degree of freedoms see [96][29][30]

$$\int d\sigma d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau) \tag{3}$$
$$X = potential\ of\ conformal\ current\ j$$

As mentioned before the quantum theory related to the Nambu-Goto action has nothing to do with its square (for more see later). On the other hand the use of the letter X for the potential of the multicomponent chiral current unfortunately suggests that Polchinski's quantum mechanical "trailer" has taken roots in the incorrect idea that the action of a multi-component massless field describes a covariant string embedded into a higher dimensional Minkowski spacetime similar to an embedding of a linear chain of oscillators into a higher dimensional QM.

The origin of this confusion is that localization as well as the probability interpretation are interpretative additions to QM by Born, whereas in LQP they are consequences of the principle of modular localization. QFT results from the quantum adaptation of the classical causal propagation principle. This leads to totally unexpected consequences: the singular nature of quantum fields (operator-valued Schwartz distributions), entropic and thermal manifestation of localized observables and the ensuing statistical mechanics like *ensemble probability* from the impure state which is the result of the restriction of the vacuum to an operator algebra $\mathcal{A}(\mathcal{O})$ of local observables, formation of vacuum polarization clouds at the causal boundary of the localization region (causal horizon) leading to an area proportionality of "localization entropy". All these phenomena are consequences of modular localization; in the more concise LQP setting of QFT in terms of nets of local operator algebras used in this paper this will be referred to as the principle of modular localization (section 3).

Global matter in QM, as e.g. the irreducible oscillator algebra which underlies chiral current models, is in certain sense "abstract" matter without causal localization. The causal localization of matter in QFT is a holistic property which results from organizing the abstract global matter according to the modular localization principle. The simplest illustration is the previously mentioned use of global creation/annihilation operators in the definition of covariant pointlike free fields which in turn generate local algebras.

It so happens that the abstract oscillator algebra underlying the 10-component supersymmetric chiral current model can be holistically organized in two different ways; one leads back to to the Möbius-covariant conformal theory localized on the lightray from which the abstract irreducible oscillator algebra was extracted in the first place. But in this special case there also happens to exist a different way which leads to a positive energy representation of the 10-dimensional Poincaré group (called supersting representation). As all positive energy representations which do not contain the so-called Wigner infinite spin representation (section 3), the *pointlike nature* of its covariant wave functions (or the associated second quantized fields) is an intrinsic consequence of energy positivity of the representation [19][20], so there is no way to relate this representation with spacetime strings. As soon as abstract quantum matter (e.g. the oscillators underlying the current algebra) are represented in order to fulfill the holistic properties of localization, the abstract matter becomes real quantum matter[10] on which the localization is imprinted.

The conformal quantum matter and the (second quantized) matter related by second quantization to the Poincare group representation share certain properties but they are not the same,; in particular there is no "embedding" of a conformal source matter into a 10 dimensional target space. The easiest way to see that the representations of this abstract algebra are different in both cases is to notice that the multi-component charge spectrum is continuous and the corresponding Poincare momentum spectrum has mass gaps.

Actually abstract matter is a fiction, it only has been invented to explain the superficial way in which ST deals with the important issue of localization. QFT with its intrinsic modular localization of quantum matter is more fundamental than global QM and its notion of intrinsically localized quantum matter has priority. Unfortunately our conceptual thinking is still dominated by QM and textbook QFT is not of much help on such issues. This makes correspondences and holographic projections very subtle issues of QFTs and causes problems in realizing that embeddings and dimensional reductions are not possible.

The X in the Polyakov action is a treacherous notation since it suggests the existence of covariant quantum mechanical string which is embedded in an n-dimensinal quantum world in an analogous way to the embedding of a classical particle. In both cases there are no covariant quantum counterparts with such properties. What really happens is that the parametrization of X does (apart from the mentioned localization point associated with the positive energy representation of the Poincaré group) not refer to spacetime but rather to an infinite dimensional inner space. This is a space "above" a localization point in which one usually pictures the spin component to reside, but which in the case at hand is infinite dimensional and which "carries" most of the infinite oscillator degrees of freedom of what was referred to as "abstract" quantum mechanical matter outside of spacetime.

It is precisely on those points on which ST and its incorrect derivatives about extra dimensions and Kaluza-Klein reductions contradicts local quantum physics. One cannot discuss problems of causal quantum localization by "massaging" Lagrangians and using quasiclassical approximations. The "heart and mind" of QFT is in the connection between phase space degrees of freedom and causal localization which is lost in such reasoning; needless to add that *there has never been a argument based on the structure of correlation functions or localized algebras* which supports such a view, despite thousands of publications on dimensional reductions, branes and similar ideas. It seems that the art to listen into a theory in oder to understand its principles and find out what it wants and what it rejects has been lost.

One unexpected remaining feature of the shared origin from the same abstract irreducible oscillator algebra is the fact that the (m^2, s) values of the superstring representation of the 10-dimensional Poincaré group is a subset of the continuous scale dimension spectrum of a 10-component chiral conformal current algebra. It consists precisely of those values which appear as the field dimensions of the composites in the global operator

[10]Actually abstract quantum matter is a fiction. Real quantum matter has its spacetime localization (including spacetime dimensions) always imprinted.

expansion of two sigma model fields. This will be mad clear in the next subsection.

This result answers a historical question asked by Majorana [38] (see below): does there exist an irreducible algebra (for the case at hand the mentioned oscillator algebra) which carries a discrete infinite component representation of the Poincaré group? The superstring representation is the positive answers to this question. The irreducible algebra is the oscillator algebra underlying the 10-component current model and the infinite component representation is the superstring representation. But the solution of this group theoretic problem is entirely kinematic in nature, it contains no informations about particle scattering or other dynamical aspects. In particular there is no relation to Mandelstam's S-matrix project in its original pre-Veneziano setting. And there is also nothing which supports an embedding picture.

The best way of presenting the group theoretical theorem discovered by string theorists is to view it, as we already indicated, in a historical context as the (presently only known) solution of the 1932 Majorana project [38]. Majorana was led to this project by the $O(4,2)$ group theoretical description of the nonrelativistic hydrogen spectrum. We take the liberty to formulate it here in a more modern terminology.

Problem 1 *(Majorana) Find an irreducible algebraic structure which carries a infinite-component positive energy one-particle representation of the Poincaré group (an "infinite component wave equation").*

Majorana's own search as well as that for the so-called "dynamic infinite component field equation" of the 60s (Fronsdal, Barut,...;see appendix of [39]) consisted in looking for irreducible group algebras of noncompact extensions of the Lorentz group ("dynamical groups"), but no acceptable solution was ever found within such a setting. The only known solution is the above superstring representation which results from an irreducible oscillator algebra of the n=10 supersymmetric Polyakov model. The positive energy property of its one-particle content and the absence of components of Wigner's "infinite spin" components (which cannot be pointlike generated) secures the pointlike localizability of this "superstring representation".

For Majorana it may have been a challenge to find an analogy of the O(4,2) hydrogen group in the context of relativistic wave functions (see below) derived from the structure of an irreducible algebra. But such a project is not supported by modern particle theory, in fact not even ST, which according to its own understanding is an S-matrix theory of interacting particles should not have any use for this group theoretic theorem; to construct an S-matrix one needs more than just group theory. The rarity of a result does not justify to use it as a metaphoric start for a new theory and enforce its recognition at conferences and in journals publications by without coming up with any tangible physical result (up to this date). This would not have happened if intelligent people with considerable reputation and charisma did not forget the important role of a critical revision of results in an increasingly speculative science as particle physics.

Problems which only seem to have one solution are especially hard to assess because the since Einstein successful method to look for an explanation in terms of an underlying principle does not apply (a unique "theory of everything" does not need an understanding in terms of principles). In such cases one sometimes obtains a better understanding by *generalizing the problem so the unicity of its solution is lost.* For example one may ask whether representations of more general noncompact groups ("noncompact inner symmetries") can live on index spaces associated to multicomponent currents, or one can extend the models to all chiral models with continuous superselected charges (in which case one would enter the largely unknow class of "nonrational" chiral models). Nothing in this direction has been done.

There is no problem to find *classical* theories with noncompact groups represented on the index spaces of their fields. Whereas for classical covariant fields this is possible, the example of the covariant classical particle Lagrangian shows that one cannot expect such a situation to have a covariant quantum counterpart. In fact the *concept of inner symmetries is a pure quantum concept* which arises from the superselection structure of the vacuum representation of observable algebras (which by definition have no inner symmetries). For QFTs with mass gaps in spacetime dimensions $d > 1+1$ this structure is discrete and leads to charge-carrying field algebras with compact group symmetries [3]. There are by now good arguments that in models with massless particles this continues to hold if one replaces the concept of superselection sector by the in this case more relevant concept of "charge class" [37]. This situation suggests that one needs chiral conformal models whose observable algebras have a continuous supply of superselection sector which limits the chiral theories (by definition) to "nonrational theories. Besides abelian current models little is known about this class.

The misunderstandings about localization is a reminder that the subtleties of the quantum causal localization principle took a long way in order to be understood. This path starts from the Einstein-Jordan conundrum and took its route through the Unruh and Hawking effects up to the recent conceptual understanding of the particle crossing property from modular wedge-localization and there is still no closure in sight.

The confusions about localization often did not enter the calculations of string theorists but remained in the interpretation. A poignant illustration is the calculation of the (graded) commutator of string fields in [40][41]. Apart from the technical problem that infinite component fields can not be tempered distribution (since the piling up of free fields over one point with ever increasing masses and spins leads to an exponentially diverging short distance scaling behavior which requires to project the string field onto finite mass subspaces), the commutator remains pointlike. Certainly this uncommon distributional behavior has no relation with the idea of spacetime strings; at most one may speak about a quantum mechanical chain of oscillators in "inner space" (over a localization point). The memory of the origin of ST from an *irreducible* oscillator algebra is imprinted in the fact that the degree of freedoms used for the representation of the Poincaré group do not exhaust the oscillator degrees of freedom, there remain degrees of freedom which interconnect the representations in the "inner" (m,s) tower. But the localization properties reside fully in these wave function spaces and, as a result of the absence of Wigner's infinite spin representations, the localization is pointlike. This is precisely what the above-mentioned authors found, but why did they not state this clearly, why they instead refer to a point on a (imagined) string? Has Heisenberg's admonition to limit quantum physics to observables been dismissed in order to serve an ideology?

Does the bizarre suggestion that we are living in an dimensionally reduced target space of an almost unique[11] 10-dimensional chiral conformal theory become more acceptable if it continues in the less bizarre but nevertheless incorrect form of embeddings of causal localizable QT i.e. in the believe that there exists a well defined geometric relation between theories of different spacetime dimensionality (embeddings and dimensional reductions)? The answer is a clear no; the ideas of Kaluza and Klein originated at a time when the foundational differences between QM and QFT were not yet noted. Such ideas may be consistent with quantum theories which do not possess an *intrinsic* notion of localization (and its subtle connection with phase space degrees of freedom) as quasiclassical approximation or QM, but they clash with the holistic aspect of modular localization which imprints the spacetime dimensionality onto causal quantum matter.

The main point of this article is to convince the unbiased reader that indeed a sizable part of the particle theory community has moved into an increasingly metaphoric direction instead of solving the hard problems of localization which existed since the time of the Einstein-Jordan conundrum and only surfaced gradually in the LQP setting of QFT. Although the errors of ST are known to most physicists familiar with the LQP who tried to understand ST from a conceptual point of view, it is not possible to overcome the present schism on this point by a rapid transfusion of LQP acquired knowledge about modular localization; the split happened already many decades ago and became solidified within globalized communities. Actually one can assign an exact date to the beginning of methaforic particle physics, it was the day of the proposal of the dual model and its subsequent widespread acceptance as an S-matrix property (see next subsection).

It is understandable that this mathematically sophisticated model had a hypnotic effect on high energy phenomenologists which at the time were looking for descriptions of infinite particle trajectories. As a result of its rich mathematical content this model also attracted more mathematical oriented physicists who thought that such deep mathematical structure deserves a connection with a more foundational kind of physics than the phenomenological "reggeology". The phenomenological excitement was cooled down after the appearance of new unsupportive observational results; but there was no critical assessment of ST on the theoretical conceptual side. To the contrary, there were comments as "ST is a gift of the 21^{th} century which by luck fell into the 20^{th} century" and similar statements by reputable physicists, and even many decades later there was no serious attempt to critically compare ST with the sucessful on-shell construction of integrable d=1+1 QFT; the few attempts to understand the origin and nature of particle crossing of S-matrices and formfactors from the causal localization principle was initially partially successful, but then got stuck in the messy details of the theory of several complex variables [14].

Res Jost was the last physicist who used his deep conceptual understanding of QFT and its relation to S-matrix properties in order to criticise the bootstrap S-matrix approach[42]. A critique of ST is more subtle and has, according to my best knowledge, never before been undertaken with the necessary conceptual mathematical precision. Part of the reason may be that the endurance of ST over so many decades is related to a somewhat confusing "picture puzzle" between (m,s) spectra of QFTs with mass gaps and (d,s) spectra in conformal QFT. In fact the solution of the Majorana project in terms of a an infinite

[11]Up to a finite number of M-theoretic modifications.

component discrete (m,s) spectrum of a 10-dimensional chiral current theory is the only known solution of this picture puzzle aspect (see below). Another reason may be that those individuals who understood the many physical and philosophical weaknesses deemed it not worthwhile to loose time in controversies with a powerful community which enjoyed considerable mathematical support. In fact up to this date only mathematicians obtained valuable informations from ideas from ST which they succeeded to make precise in a way which suits them. Part of the reason why ST was not analyzed from a foundational viewpoint (as in this article) may be related to the support from the mathematical side.

Another kind of critique amounts results from the derivation of the true particle crossing from the principle of modular localization. This does not only reveal the difference to dual model crossing, but also suggests a new on-shell construction methodes based on the S-matrix which is capable to replace Mandelstam's approach (section 4).

2.2 The picture puzzle of chiral models and particle spectra

There are two ways to see the correct mathematical-conceptual meaning of the dual model and (what for historical correctness is called) ST without being side-tracked by treacherous analogies.

One uses the "Mack machine" [17][18] for the construction of dual models (including the dual model which Veneziano constructed "by hand"). One starts from a conformal 4-point function of *any conformal QFT in any spacetime dimension.* To maintain simplicity, we take the vacuum expectation of four (not necessarily equal) scalar fields

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \tag{4}$$

It is one of the specialities of interacting *conformal* theories that fields have no associated particles, instead they carry a (generally a non-canonical, anomalous) scale dimensions which is connected with the nontrivial *center of the conformal covering group* [10]. It is well known from the pre BPZ [43] conformal research in the 70s [44] [45] that conformal theories have converging operator expansions of the type

$$A_3(x_3)A_4(x_4)\Omega = \sum_k \int d^4z \Delta_{A_3,A_4.,C_k}(x_1,x_2,y)C_k(z)\Omega \tag{5}$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \rightarrow 3\ different\ expansions \tag{6}$$

In distinction to the Wilson-Zimmermann short distance expansions which only converge in an asymptotic sense, these expansions converge in the sense of vector-valued Schwartz distributions. The form of the global 3-point-like expansion coefficients is completely fixed in terms of the anomalous scale dimension spectrum of the participating conformal fields; i.e. unlike in models with a particle interpretation, one does not have to dive deeply into the dynamics in order to get a rather explicit understanding of the operator expansions and their coefficient functions.

It is clear that there are exactly three ways of applying global operator expansions to pairs of operators inside a 4-point-function 6, analogous to the three possible particle pairings in the elastic S-matrix which correspond to the s,t and u in Mandelstam's formulation of crossing. But beware, this dual model crossing arising from the Mellin transform of conformal correlation has no relation with S-matrix particle crossing in Mandelstam's on-shell project! If duality would have arisen in this this conformal context probably nobody would have connected it with the particle crossing in S-matrices and on-shell formfactors. But Veneziano found it [16][46] from properties of the Euler beta function which did not reveal its conformal origin. Since particle crossing and its conceptual origin in the principles of QFT remained somewhat hidden (for a recent account of its origin from modular localization see [47][10]) the identification of crossing with Veneziano's duality met little resistance. As mentioned it could have been clear with a bit more hidsight that it has no relation to particle crossing since the S-matrix *cannot be meromorphic* in Mandelstam's variables (and cannot even be approximated in this way); many useful messages to this extend could already have been learned from the rigorous construction of integrable models which have no inelastic processes; their scattering functions are meromorphic in the rapidity uniformization variables but not in Mandelstam's s,t,u.

The Mellin transform of the 4-point-function is a meromorphic function in s,t,u (with appologies for the in this case treacherous notation). It has first order poles at the numerical values of the anomalous dimensions of those conformal composites which appear in the three different global decompositions of products of conformal fields; they are related by analytic continuation [17][18]. To enforce an interpretation of particle masses, one may rescale these dimensionless numbers by the same dimensionfull number. However this formal step of calling the scale dimensions of composites particle masses does not change the physical reality. Structural analogies in particle physics are worthless without an independent support concerning their physical origin.

The Mack machine to produce dual models (crossing symmetric analytic functions of 3 variables) has no definite relation to spacetime dimensions; one may start from a *conformal theory in any spacetime dimension* and end with a meromorphic crossing function in Mellin variables. Calling them Mandelstam variables does not change the conceptual-mathematical reality which for scattering amplitudes (unitarity, inconsistency of particle crossing which are meromorphic in Mandelstam variables) is totally different from that of Mellin transforms of conformal correlation. One is dealing with two objects whose position in Hilbert space which could hardly be more different than that scattering amplitudes and conformal correlations; no unitarization scheme can mathematically change one into the other.

However, and here we come to the *picture-puzzle aspect* of ST, one can ask the more modest question whether one can view the *dimensional spectrum of composites* in global operator expansions (after multiplication with a common dimensionfull $[m^2]$ parameter) *as arising from a positive energy representation of the Poincaré group.* The only such possibility which was found is the previously mentioned 10 component superymmetric chiral current theory which leads to the well-known superstring representation of the Poincaré group and constitutes the only known solution of the Majorana project[12]. In this way the analogy of the anomalous composite dimensions of the poles in the dual model from the Mack machine to a m^2 mass spectrum is extended to a genuine particle representation of the Poincaré group in terms of masses which are proportional to a

[12]To see this, the representation theory of the irreducible oscillator algebra of the chiral current model is more suitable than the Mack machine.

conformal sub-spectrum. But even this lucky circumstance which led to the superstring representation remains on the level of group theory and cannot be viewed as containing dynamic informations about a scattering amplitude; not even in an approximate sense!

There exists a presentation which exposes this "picture-puzzle" aspect between conformal chiral current models and particle properties in a stronger way: the so-called sigma-model representation. Schematically it can be described in terms of the following manipulation on abelian chiral currents ($x =$ lightray coordinate)

$$\begin{aligned}
&\partial\Phi_k(x) = j_k(x),\ \Phi_k(x) = \int_{-\infty}^{x} j_k(x),\ \ \langle j_k(x)j_l(x')\rangle \sim \delta_{k,l}\left(x - x' - i\varepsilon\right)^{-2} \qquad (7)\\
&Q_k = \Phi_k(\infty)\,,\ \Psi(x,\vec{q}) = \text{"}:e^{i\vec{q}\vec{\Phi}(x)}\text{"}:\ ,\ carries\ \vec{q}-charge\\
&Q_k \simeq P_k,\ dim(e^{i\vec{q}\vec{\Phi}(x)}) \sim \vec{q}\cdot\vec{q} \simeq p_\mu p^\mu,\ (d_{sd}, s) \sim (m, s)
\end{aligned}$$

The first line defines the *potentials of the current*; it is formally infrared-divergent and should not be used to generate the vacuum sector which is created from the vacuum by applying the polynomial algebra generated by the current alone. In contrast *the exponential sigma field* Ψ *is the formal expression for a covariant superselected charge-carrying field* whose symbolic exponential way of writing leads to the *correct correlation functions only in total charge zero correlations* where the correlation functions agree with those computed from Wick-reordering[13] of products of : $expiq\Phi(x)$:,all other correlations of the sigma-model field vanish (indicated by the quotation mark which indicates this limitation of the formal notation).

The interesting line is the third in (7), since it expresses a "mock relation" with particle physics in which the multi-component continuous charge spectrum of the conformal currents resemble the continuous momentum spectrum of a representation of the Poincaré group, whereas the spectrum of anomalous scale dimensions (being quadratic in the charges) is reminiscent the quadratic relation between momenta and particle masses. The above analogy amounts to a genuine positive energy representation of the Poincaré group for the special case of a supersymmetric 10-component chiral current model; it is the before-mentioned solution of the Majorana project; but its appearance in the Mellin transform of a conformal correlation has nothing to do with an S-matrix. As also mentioned, the shared irreducible abstract oscillator algebra leads to different representations in the use for the conformal theory and the localization which is related to the positive energy representation of the Poincaré group[14]. The difference between the representation leading to the conformal chiral theory and that of the Poincare group on the target space (the superstring representation) prevents the (structurally anyhow impossible) interpretation in terms of an embedding of QFTs, although there remains a certain closeness as a result of the shared oscillator algebra.

The multicomponent Q_μ charge spectrum covers the full $\mathbb{R}^{10}$ whereas the P_μ spectrum of the superstring representation is concentrated on positive mass hyperboloids; the

[13]The two-point function of Φ being the indefinite metric logarithm of x-x'.is indefinite but the exponential correlations together with the charge-conservation coply with the Hilbert space structure.

[14]The 26 component model does not appear here because we are interested in localizable representation; only positive energy representations are localizable.

Hilbert space representation of the algebraic oscillator substrate in order to obtain localization and Möbius invariance on the light ray is not the same as that which leads to that of the superstring representation. Hence presenting the result as an embedding of the chiral "source theory" into the 10 component "target theory" is a metaphoric exaggeration having its psychological origin probably in the picture-puzzle aspect; the representation theoretical differences express the different holistic character of the two different localizations (the target localization being a direct consequence of the intrinsic localization of positive energy representations of the Poincaré group). What remains is a mathematical question: why does the positive energy representation of the Poincaré group only occur when the chiral realization has a vanishing Virasoro algebra parameter? And are there other non-rational (continuous set of superselection sectors) chiral models which solve the Majorana project? Both questions can be generalized to: are their other nonrational chiral theories with (discrete sums of irreducible) representations of inner noncompact symmetries (target representations) ?

It should be added that it would be totally misleading to reduce the mathematical/conceptual role of chiral abelian current models to their picture puzzle use in ST or their role in the solution of the Majorana project. The chiral n-component current models played an important conceptual role in mathematical physics; the so-called *maximal extensions* of these observable algebras can be classified by even integer lattices and the possible superselection sectors of these so extended algebras are classified in terms of their duals [48][49][50]. Interestingly the selfdual lattices and their relation with exceptional final groups correspond precisely to the absence of non-vacuum superselection sectors which in turn is equivalent to the validity of *full* Haag duality (Haag duality also for all multiply-connected algebras [33][34]). They constitute the most explicitly constructed nontrivial chiral models which shed light on the interplay of discrete group theory and Haag duality as well as on violation of Haag duality for disconnected localization regions and anyon statistics as well as many other surprising consequences of modular localization. This more than a consolation for their inability to reveal properties about higher dimensional scattering amplitudes.

2.3 General structural arguments against embeddings and dimensional reductions

The important property which permitted to associate the representation of a noncompact group (in the above case the Poincaré group) with the "target" of a chiral QFT, is the existence of superselected charges with a continuous spectrum. This is only possible in chiral theories, more specifically in nonrational (by definition) models, the only known case is the one presented in the previous subsection.

This cannot occur in a higher dimensional theory with a complete particle interpretation since the DHR superselection theory (and its Buchholz-Fredenhagen [51] extension) leads to a inner compact group [3]. The inner symmetry idea, which dates back to Heisenberg's isospin, is a quantum notion which in the quantization approach to QFT is "red back" into the classical Lagrangian field formalism; classical Lagrangians can also support the action of noncompact symmetry groups. A trivial example is the mentioned classical covariant path Lagrangian $\sqrt{ds^2}$ whose Euler-Lagrange equation is the covariant description of a classical particle which has no quantum counterpart, and whether the classical covariant surface solutions of the N-G Lagrangian admit a covariant quantum counterpart is still questionable. Quantization is not a principle rather it is a conceptional limited but observational successful artistic device; not every covariant classical theory has a covariant quantum counterpart, neither can one expect that a QFT, which has been constructed in an intrinsic way (see the algebraic construction in section 4), can be described in terms of Lagrangian quantization.

The concept of causal localization is too holistic in order to permit an embedding or a dimensional reduction outside of quasiclassical approximations of QFT, it would contradict the principle of modular localization. Nobody has ever been able to show that the correlation functions of a model in lesser spacetime dimension can be obtained by restricting a QFT, nor that the inverse association of two QFTs by embedding is possible. The lack of any intrinsic structural argument (which according to the modular localization aspect of LQP cannot exist) did not prevent the appearance of thousands of papers and the creation of special sections in journals and at conferences. This has grown into a sociological/psychological bulwark which seems to be impenetrable to scientific arguments. Existing "proofs" of the Kaluza-Klein mechanism in QFT are always based on massaging Lagrangians or manipulations in terms of quasiclassical approximations but such arguments ignore what happens with genuine quantum degrees of freedom in such manipulations. A closely related issue is that of *branes*; in that case Mack [17][18] has shown that in passing from the full theory to a brane, there is *no thinning out of degrees of freedom*. This preservation of cardinality of phase space degrees of freedom leads to an non acceptable causality violation (violation of the causal completeness property, the "poltergeist phenomenon").

This causality violation is the same as that occurring in the mathematical AdS-CFT correspondence. If one starts from a locally causal AdS model, the associated CFT will be unphysical as a result of that poltergeist-causing violation of causal completeness; and in case one starts from a physical CFT, the resulting AdS model whose existence is guaranteed by the correspondence will be too "anemic". In fact its compact double cone algebras has no degrees of freedom (i.e. are trivial) and only in noncompact spacetime regions extending to infinity (as wedges) degrees of freedom will be present [52]. The Maldacena conjecture, which presumes that both sides of the correspondence are "physical", plainly contradicts these facts.

Observations about the relation between the independence of the causal completeness property from the Einstein causality started in the early 60s [28]; the use of generalized free fields also indicated the relation with too many degrees of freedom. Later this connection between the cardinality of degrees of freedom and causal localization was sharpened first to compactness and afterwords to "nuclearity" [3].

2.4 The correct implementation of quantization for the N-G action

The classical geometric surface embedding as defined by the N-G action and treated according to the rules of reparametrization-invariant quantum systems poses a similar problem of diffeomorphism invariance as the quantization of the Einstein-Hilbert action

[53]. This is intimately connected with the physical problem of implementing *background independence* in both of these cases. Even though the E-H and the N-G actions are non-renormalizable (i.e. its perturbative calculations leads to an increasing with perturbative order set of undetermined parameters), there are arguments that the issue of background independence can be discussed independent of the renormalizability issue [54] (in which case the principle of background independence cannot be used to restrict the increasing number of parameters .

The problems of the N-G quantization and its diffeomorphism covariance has been recently treated in [53]. The application of this computational setting to the $\sqrt{ds^2}$action results apparently in the quantum theory of a nonrelativistic particle[15] and there is no reason that its N-G counterpart has anything to do with a *covariant* QT in the sense of a representation of the Poincaré group. So it seems that only the canonical quantization of the Polyakov action can be associated with a representation of the Poincaré group which solves, as explained before, the Majorana project, but has no relation to an on-shell construction of the kind Mandelstam was looking for.

It is hardly to imagine that string theorists would be satisfied with such a group theoretic result, but it is the only mathematical fact which can be salvaged from the Mandelstam on-shell construction project after the incorporation of Veneziano's dual model and ST. In the next section we will present the derivation of particle crossing from the modular properties of wedge-localization. This does not only show that there is no relation to the crossing used in ST, but it also leads to a new formulation of an on-shell construction project which may be considered as a extension of what Mandelstam had in mind before the appearance of the dual model.

Before closing this subsection it may be interesting to mention another more concrete attempt to explore the physical content of the quantum N-G model. This is due to Pohlmeyer [55] who established the *existence of infinitely many classical conservation laws,* which suggest that the model is integrable. For integrable models there exists a more intrinsic way of quantization which is based on the Poisson bracket structure between the globally conserved quantities. Such a quantization has a higher degree of plausibility than canonical quantizations (which anyhow do not refer to the N-G action but rather to its Polyakov square). In a series of high-quality publications Pohlmeyer and his collaborators studied the quantization of the Poisson-bracket relations between the conserved quantities. The drawback from the point of view of intention of the ST community is that this does not reveal anything about local observables and their possible covariant behavior under Poincaré transformation.

Since these were rigorous, albeit incomplete mathematical results about the quantum theory of the N-G action, Pohlmeyer also called his approach to this model string theory. So the reader should be aware of the multifaceted use of "string" in Pohlmeyer's infinite set of conservation laws abstracted from the N-G action, the [53] quantization of the N-G action according to the rules for diffeomorphic actions and the canonical quantization of the Polyakov action (the strings of ST); these two actions lead to different quantum theories even though there is a correspondence between some of their classical solutions. In all those cases the use of "string" refers to different properties of the relativistic classical

[15] I am indebted to Jochen Zahn for informing me on this point.

action (traced out world sheets instead of world lines) but in none of these cases this has anything to do with a string localized in spacetime.

After having explained why the content of ST is entirely group theoretical and can be best stated by saying that the abstract irreducible oscillator algebra underlying the 10-component Polyakov action has (at least) two inequivalent representations, one which is organized according to the holistic properties for conformal localization on a lightray and the other which represents a positive energy discrete representation of the Poincaré group which realizes the holistic "target" localization, the reader may be curious to learn *how genuine string-localized objects look really like* and what, if any, is their expected physical role in particle theory. This will be the content of the next section.

3 Higher spin interactions and modular localization

In this section we try to familiarize the reader with the concept of modular localization and its use in passing from certain nonrenormalizable interactions in terms of pointlike fields to renormalizable couplings for stringlike fields. In the first subsection we start with a short review about the connection between positive energy representations of the Poincaré group and the construction of point-localized covariant fields similar to Weinberg's method of covariantizing Wigner's representations [56]. This subsection also introduces a pedestrian description of string-localized free fields as well as a schematic description of their ongoing use in the enlargement of renormalizability to interactions involving arbitrary high spins.

A conceptual/mathematical backup in terms of modular localization is the task of the second subsection, whereas the third subsection is meant to indicate the enormous potential of these ideas in the ongoing and future research.

3.1 Wigner representations and their covariantization

Historically the use the new setting of modular localization started with a challenge left over since the days of Wigner's particle classification: *the causal localization of the third Wigner class* (the massless infinite spin class) of positive energy representations of the Poincaré group. Whereas the massive class as well as the zero-mass finite helicity class are pointlike generated, it is not possible to find covariant pointlike generating wave functions for this third Wigner class. The first representation theoretical argument suggesting the impossibility of a pointlike generation dates back to [57]. It was followed decades later by the concept of modular localization of wave functions [19][22] which led to the introduction of spacelike string-generated fields in [20]. These are covariant fields $\Psi(x,e)$, e spacelike unit vector, which are localized $x+\mathbb{R}_+e$ in the sense that the (graded) commutator vanishes if the complete semiinfinite strings (and not only their starting points x) are spacelike separated [20]

$$[\Psi(x,e),\Phi(x',e')]_{grad}=0,\ x+\mathbb{R}_+e\ \rangle\langle\ x'+\mathbb{R}_+e' \qquad (8)$$

Unlike decomposable stringlike fields (pointlike fields integrated along spacelike halflines) such *elementary stringlike fields* lead to serious problems with respect to the activation of (compactly localized) particle counters. The decomposable strings of higher spin potentials (see next section) are in an appropriate sense "milder". As always, free string-localized fields are characterized by the property that their Fourier-transforms are on-shell.

In the old days [56] infinite spin representations were rejected on the ground that nature does not make use of them. But whether in more recent times of dark matter one would uphold such dismissals is questionable. String-localized quantum fields fluctuate both in x as well as in e[16]. They can always be constructed in such a way that their effective short distance dimension is the lowest possible one allowed by positivity, namely $d_{sd} = 1$ *for all spins*.It is very difficult to construct the covariant "infinite spin" fields by the group theoretic intertwiner method used by Weinberg [56]; in [20][58] the more powerful setting of modular localization was used. In this way also the higher spin string-localized fields were constructed.

For finite spins the unique Wigner representation always has many covariant pointlike realizations, in fact there are many pointlike spinorial descriptions associated to one Wigner representation; the associated quantum fields define linear covariant generators of the system of localized operator algebras whereas their Wick powers are nonlinear. We now explain the reasons why even in case of pointlike generation on is interested in stringlike generating fields [20].

For pointlike generating covariant fields $\Psi^{(A,\dot{B})}(x)$ one finds the following possibilities which link the physical spin s to the (undotted, dotted) spinorial indices

$$\left|A - \dot{B}\right| \leqslant s \leqslant A + \dot{B},\ m > 0 \tag{9}$$

$$h = A - \dot{B},\ m = 0 \tag{10}$$

In the massive case all possibilities for the angular decomposition of two spinorial indices are allowed, whereas in the massless case the values of the helicities h are severely restricted (second line). For $(m = 0, h = 1)$ the formula conveys the impossibility of reconciling pointlike vector potentials with the Hilbert space positivity. This clash holds for all $(m = 0, s \geq 0)$: pointlike localized "field strengths" (in h=2, the linearized Riemann tensor,..) have no pointlike quantum "potentials" (in h=2, the $g_{\mu\nu}$,...) and similar statement holds for half-integer spins in case of $s > 1/2$. Allowing stringlike generators the possibilities of massless spinoral $A, \dot{B}$ realizations are identical to those in the first line (9).

Since the classical theory does not care about positivity, (Lagrangian) quantization inevitably forces the *abandonment of the Hilbert space in favor of Krein spaces* (implemented by the Gupta-Bleuler or BRST formalism). The more intrinsic Wigner representation-theoretical approach keeps the Hilbert space and lifts the restriction to pointlike in favor of semiinfinite stringlike generating fields.

It is worthwhile to point out that perturbation theory does not require the validity of Lagrangian/functional quantization. Actions which lead to Euler Lagrange quantization limit the covariant realizations of (m,s) Wigner representations to a few spinorial/tensorial fields with low $(A, \dot{B})$ but as Weinberg already emphasized for setting up perturbation theory one does not need Euler-Lagrange equations; they are only necessary if one uses

[16] These long distance (infrared) fluctuations are short distance fluctuation in the sense of the asymptotically associated d=1+2 de Sitter spacetime.

formulation in which the interaction-free part of the Lagrangian enters as in the Lagrangian/functional quantization. The only "classical" input into causal perturbation as the E-G approach is a (Wick-ordered) polynomial which implements the classical pointlike coupling, all subsequent inductive steps use quantum causality. In the modular localization based setting of section 3 even this last weak link with classical thinking is cut and one enters the area of LQP without classical crutches.

For $(m = 0, s = 1)$ the stringlike covariant potentials $A_\mu(x, e)$ are uniquely determined in terms of the field strength $F_{\mu\nu}(x)$ and a spacelike direction e. The idea is somewhat related to Mandelstam's early attempt to formulate QED without the vectorpotentials [15]. But even though the string-local potential is uniquely determined in terms of $F_{\mu\nu}$ and e , it is much safer to explicitly introduce the covariant $A_\mu(x, e)$ (represented as a semiinfinite integral over the field strength along a semiinfinite line in the direction e) as an object in its own right because in this way one cannot overlook that one is dealing with *objects which fluctuate in both x and e*; in fact the improvement of the short distance property in x is paid for by a worsening but still well-defined infrared behavior i.e. the $A_\mu(x, e)$ is an *operator-valued distribution in both x, e*. In contrast to the above infinite spin representation which cannot be pointlike generated, all other zero mass representations admit pointlike generators and only exclude pointlike potentials.

As an illustrative example for the use of those objects, let us look at the Aharonov-Bohm effect in QFT[17]. In terms of QFT in the LQP formulation this amounts to a breakdown of *Haag duality* (2) *for a toroidal spacetime localization* [33][34]

$$\mathcal{A}(\mathcal{T}') \subsetneq \mathcal{A}(\mathcal{T})' \tag{11}$$
$$\mathcal{T}\ \ \textit{spatial torus at } t = 0\ ,\ \mathcal{T}\,''\ \textit{its causal completion}$$

For lower spin zero mass fields or for a torus-localized algebra from a massive field of any spin, one finds the equality sign (Haag duality). This can be shown in terms of field strengths, but if one (for the convenience of applying Stokes theorem) uses the indefinite metric potentials one gets the wrong result, namely equality (zero effect). On the other hand the use of the string-localized potential in the Hilbert space accounts correctly for the A-B effect as the breakdown of Haag duality for multiply connected spacetime regions. It is expected that the breakdown of Haag duality for multiply connected regions is a general feature of higher spin zero mass representations.

In massive theories there is no such clash between localization and Hilbert space and there is also no violation of Haag duality in multiply connected regions. Pointlike potentials exist in Hilbert space (e.g. the Proca vectorpotential), but their short distance dimensions increase with spin just like those of field strengths (example: $d(A_\mu^P) = 2$). Nevertheless one can introduce *stringlike potentials as a means to lower the short distance dimension* in order make the couplings fit for renormalization. The connection between the covariant[18] stringlike vectorpotential and its pointlike counterpart (the $d_{sd} = 2$ *Proca* potential $A_\mu^P(x)$) leads to a scalar string-localized field, the counterpart of the Stückelberg field

$$A_\mu(x, e) = A_\mu^P(x) + \partial_\mu \phi(x, e) \tag{12}$$

[17]The standard A-B effect is about quantum mechanical charged particle in an *external* magnetic field.
[18]The spacelike string-direction e participates as a vector in the covariance law.

Note that here the scalar Stückelberg field is not an independent field (as it would be in the BRST setting) i.e. the string description of free fields has mixed two-point functions between $A_\mu(x,e)$ and $\phi(x,e)$; the physical Hilbert space in which also the string-localized fields are living is determined in terms of $A_\mu^P(x)$ and the e-dependent covariant fields are relatively string-local (same Borchers class) with respect to the Proca field[19].

The strategy of the implementation adiabatic equivalence starts with the zero order relation (12) which is used in the Bogoliubov formula for the perturbative physical S-Matrix and the physical fields. For massive QED the interaction density $\mathcal{L}$

$$\mathcal{L}(x,f) = \int def(e)\mathcal{L}(x,e),\ \mathcal{L}(x,e) = A_\mu(x,e)j^\mu(x),\ \mathcal{L} = \int g(x)\mathcal{L}(x,f)dx \qquad (13)$$
$$\psi_{int}(x,f) := \frac{\delta}{i\delta h(x)}S(\mathcal{L})^{-1}S(\mathcal{L}+h\psi)|_{h=0},\ S(\mathcal{L}) = Te^{i\int g(x)\mathcal{L}(x,f)dx}$$

leads, according to the formal Bogoliubov prescription, to the perturbative S-matrix as well as to fields indicated for the simplest case in the second line for the interacting Dirac spinor; time-ordered products of interacting products originate from higher functional derivatives[20]. The physical S-matrix results from the Bogoliubov S-functional in the adiabatic limit $g(x) \equiv 1$. The existence of this limit is only guarantied in the presence of mass gaps. The physical interacting fields $\psi_{int}^{phys}(x,f)$ also require this adiabatic limit; but as a result of the appearance of the inverse S-functional, the requirement for their existence are less stringent. They are localized in a spacelike cone with apex x and require the same renormalization treatment as a pointlike d=1 field.

The reason why the smearing function in the string direction can be fixed, is that it plays a different role from g since no limit has to be performed on them. The resulting physical $\psi(x,f)$-field depends nonlinearly on f and is localized in a spacelike cone with apex at x[21], but whose distributional extension problem still follows the iterative E-G scheme in which only the remaining counterterm liberty is still determined by the total diagonal in the apices [63]. The physical content of the theory can be extracted from the spacelike cone localized fields for fixed f-smearing since the LQP description of particle physics also works for spacelike cone localization [3].

The important aspect to notice in connection with string-localized fields with finite spin is that their Wigner particle representations always admit covariant potentials which have the lowest possible x-singularity which accounts for their short distance dimension d=1 and thus permits. In contrast to pointlike realizations they achieve this improved short distance behavior by "spreading" the difference between the d_{point} which increases with s to $d_{string} = 1$ "over the string" which accounts for the fact that, although the string localization is seen in the commutation relations (8) of the potentials, the counterterm freedom of E-G renormalization is still described by pointlike terms.

In fact the main new idea used in the ongoing research on this problem is that certain formally pointlike nonrenormalizable couplings (e.g. interactions involving massive vector-potentials) are "adiabatically equivalent" to renormalizable stringlike formulations. This important new insight also amounts to the possibility to compute a point-like perturbation

[19] If one reads the equation as a definition of A^P, one easily shows $d_e(A_\mu - \partial_\mu\phi) = 0$

[20] In order to include field strengths one needs another source term i.e. $S(L + h\psi + kF)$.

[21] The apex is also the point which is relevant for the Epstein-Glaser distributional continuation.

series via the round-about way of doing renormalization theory in the string-like setting and afterwords passing via adiabatic equivalence to the pointlike correlation functions. In this way an old idea [60] to go beyond Schwartz distributions to what is now known as "hyperfunctions" in order to enter the area of nonrenormalizability takes on new actuality. Mathematicians found an interesting subset of hyperfunctions which still admit dense sets of compactly localizable test functions [61] which Arthur Jaffe [62] introduced as "strictly localizable fields" (SLF) into QFT, showing among other things that the exponential of a free field and some of those fields used in couplings between massive vectorpotentials and scalar fields in [60] belong to this class. At the time when these distributional extension of singular fields were proposed, there was no motivating application for their introduction; nonrenormalizable theories did not become less nonrenormalizable as a result to this SLF extension. As a result of the newly discovered principle of adiabatic equivalence, which in the case at hand means that massive nonrenormalizable couplings of pointlike d=2 Proca vectormesons can be converted into d=1 renormalizable stringlike vectormesons with the high-dimensional contributions being absorbed into surface harmless surface terms, the concept of nonrenormalizability undergoes a radical change.

A direct formulation in terms of pointlike fields would still result in the appearance of infinitely many undetermined counterterms, *but doing the renormalization in terms of stringlocal potentials and then passing to the pointlike description shows that this kind of pointlike nonrenormalizability maintains the same finite number of parameters as the renormalizable stringlike description*; in fact both fields are in the same local equivalence (Borchers-) class. The principle of modular localization opens the gates for renormalizability for higher spins!

Taking into account the short-distance scaling degree of free massive string-localized potentials $d_{string} = 1$ instead of $d_{point} = 2$ for pointlike potentials (Proca), the formulation of the adiabatic equivalence principle starts with establishing the following first and second order relation ($\mathcal{L}^P$ Proca Lagrangian $d_{s.d.} = 5$)

$$\begin{aligned} &\mathcal{L} = \mathcal{L}^P + \partial_\mu V^\mu,\ V^\mu(x,e) = j^\mu(x)\phi(x,e),\ \mathcal{L}' \equiv \mathcal{L}(x_2,e_2) \\ &T\mathcal{L}\ \mathcal{L}' - \partial_\mu T\ V^\mu \mathcal{L}' - \partial_\nu' T\mathcal{L}\ V^{\nu\prime} + \partial_\mu \partial_\nu' T V^\mu V^{\nu\prime} = T\mathcal{L}^P \mathcal{L}^{P\prime} \end{aligned} \tag{14}$$

The last relation is a formal second order relation between the string- and point-like description. It is trivially satisfied for the first term in the Wick expansion of time-ordered products. It is reasonably easy to check in the tree approximation. To fulfill it in the loop term is more demanding. The pointlike nature of the tree and loop terms is established by showing that the directional derivative d_e and $d_{e'}$ vanish[22]. For details we refer to a forthcoming paper by Jens Mund [63]. The fact that scalar "massive QED" has quadratic terms in the vectorpotentials changes the situation somewhat, but the adiabatic equivalence with the nonrenormalizable pointlike formulation also goes through [64]. Note that the crucial point of the adiabatic equivalence is that the [23]difference between the nonrenormalizable pointlike and the renormalizable stringlike formulation consists of derivative terms which vanish in the adiabatic limit; the high dimensional

[22]The E-G extension of the loop term is quite tricky, but the resulting counterterm freedom is still pointlike [63].

[23]Part of joint project together with Jens Mund and Jakob Yngvason.

terms which rendered the pointlike formulation nonrenormalizable are flushed away to infinity.

New problems however arise in Yang-Mills couplings as a consequence that the equation which prepares the implementation of the adiabatic equivalence become nonlinear in higher orders (color indices omitted)

$$A_\mu(x,e) = U(\phi(x,e))A_\mu^P(x)U(\phi(x,e))^* + \partial_\mu\phi(x,e) \tag{15}$$

Here the color components of $\phi(x,e)$ multiplied with the coupling function g play the role of the numerical parameters in the U-color rotation.

It is conceivable that this already contributes to the second order in addition to those contributions which come from the trilinear and quadrilinear selfinteractions of the vectorpotentials. It would be desirable to show that the U are exponential SLF fields in the sense of Jaffe, but this is not necessary for the perturbative use of (15) for the implementation of the adiabatic äquivalence. This calculation, which may decide over whether there is a theoretical necessity for the coupling to neutral scalar multiplets (Higgs) fields, has not been done at the time of writing of the present paper.

There exists another approach to massive vectormeson coupling, the BRST formalism. The formula which relates the Proca potential to a dimension d=1 BRST potential is similar to the above

$$A_\mu^{BRST}(x) = A_\mu^P(x) + \partial_\mu\phi^{BRST}(x) \tag{16}$$

where ϕ^{BRST} is the indefinite metric Stückelberg field. In this case the lowering of $d_{s.d.}$ from 2 to 1 is the result of the Krein space setting. But whereas in the string-localized description the matter-field exists as a spacelike cone-localized object which is still a distribution in x, the existing literature does not contain a prescription for obtaining physical matter fields within the BRST formalism but is presently restricted to the S-matrix [65] of massive vectormesons. Therefore it is interesting to note that the application of the adiabatic equivalence suggests a way to solve this open problem.

After having addressed technical question concerning the renormalization process for abelian massive vectormesons, there remains the important question of the existence of pointlike physical fields. To be more explicit, this amounts in both cases (string-localization and the BRST formulation in Krein space) to the question of the status of a pointlike matter-field within a formally nonrenormalizable coupling. The only requirement on such a field is that it is relative local to the string-localized respectively pointlike BRST fields. The existence of such singular but yet localizable objects is strongly suggested[24] by the following *adiabatic equivalence* relation for the generating Bogoliubov S-functional

$$\begin{aligned}
&S(\mathcal{L} + h\psi + kF) \simeq S(\hat{\mathcal{L}} + h\hat{\psi} + kF) \\
&\hat{\mathcal{L}} = \mathcal{L}(A_\mu(x,e) \to A_\mu^P(x)),\ \hat{\psi}(x) = e^{-i\phi(x,f)}\psi(x,f) \\
&or\ for\ BRST:\ \hat{\psi}(x) = e^{-i\phi(x)}\psi^{BRST}(x),\ if\ \mathcal{L} \to \mathcal{L}^{BRST}
\end{aligned} \tag{17}$$

where the $\simeq$ stands for equality in the adiabatic limit $g(x) \to 1$. The transformation to the new S-functional is a formal operator gauge transformation inside the same theory which

[24]Pointlike matter-fields with a bad high energy behavior already appeared in the "unitary gauge" of the oldest results about massive QED [66].

leaves F unchanged and transforms ψ into the (nonrenormalizable) pointlike candidate $\hat{\psi}$ for the the matter field in the adiabatic limit. The adiabatic equivalence of the BRST functional with that formally obtained from the nonrenormalizable pointlike formulation in a Hilbert space is the corresponding statement in case of the Krein formulation; in this case all fields remain formally point-localized. But whereas the string-like formulation allows a massless limit, *the pointlike BRST formulation has no massless physical* $\hat{\psi}$ *limit* (there are simply no pointlike fields to which stringlike fields could be adiabatically equivalent), there are simply no pointlike potentials (A_μ^P has no massless limit) as a result of the clash between pointlike localization and Hilbert space.

Since the string-localized massless vectorpotentials of the Hilbert space formulation are uniquely fixed in terms of the field strength $F_{\mu\nu}(x)$ and the spacelike string direction e, the input is the same as in Mandelstam's attempts to formulate QED solely in terms of field strengths. It turns out that precisely the directional fluctuation of the $x + \mathbb{R}_+ e$ localized $A_\mu(x, e)$ in e (a point in d=1+2 de Sitter spacetime) attenuate the strength of the x-fluctuations and renders the interaction renormalizable in the sense of power-counting. The picture is that the nonvanishing commutators for string crossing are necessary for lowering the singularity for coalescent x. Mandelstam's approach probably failed because in his setting it seems to be difficult to take care of this advantage [59]. In both, the massless as well as the massive case, there always exists a string-localized description in which the e-fluctations lower the strength of the x-fluctuation in the pointlike description in such a way that the resulting short distance scale dimension is d=1 independent of spin.

This deeper understanding, which is unfortunately blurred behind the widespread accepted vernacular "long distances are outside of perturbation theory"[25], leads to the recognition that the correctly formulated massless perturbation approach using the stringlike nature of fields avoids these off-shell infrared divergence problems in the standard formulation of Yang-Mills couplings. The only remaining genuine infrared problem is the question of how to relate perturbatively well-defined string-localized correlation functions to measured charged particles without the infinite infrared clouds of photons entering the large time asymptotic behavior [37].

In this aspect the stringlike Hilbert space formulation is superior to the Krein space formulation. It presents for the first time a *rigorous perturbative way* to check the asymptotic freedom statements based on the mass-independent beta function within a formulation with perturbative well-defined Callen-Symanzik equation. In general the pointlike fields which appear in the adiabatic equivalence relation of massive vectormeson models constitute a perturbative construct since the pointlike Hilbert space formalism is not renormalizable. In fact they have a good chance to be SLF (strictly localizable) fields in the sense of Jaffe [62] which, although not being Schwartz distributions as a result of their bad short distance properties, are still localizable. The pointlike Hilbert space field $\hat{\psi}$ in (17) is connected with its renormalizable BRST or string-localized counterpart by an operator gauge transformation in terms of an exponential in the Stückelberg field which makes the physical spinor field nonrenormalizable but maintains its affiliation to a

[25]The infrared-finiteness of correlations in the string-localized Hilbert space description shows that the infrared divergences in the standard approach are a result of the of the nonexistence of physical pointlike fields in the massless limit.

finite-parametric QFT[26].

The advantage of using string-localized fields in Hilbert space instead of pointlike fields in Krein shows up most forcefully when higher spin zero mass fields participate in the interaction. In that case pointlike objects which are analogous to Proca fields simply do not exist, which makes it impossible to describe e.g. massless gauge theories in a covariant pointlike Hilbert space setting. In other words pointlike physical descriptions obtained in the Krein space setting have genuine infrared divergences since in that limit. There is also the problem of the physical credibility of the result that the consistency of the renormalization of *massive Y-M in the Krein setting* requires the presence of scalar multiplets; this is a problem which is independent of the metaphoric picture about the Higgs mechanism [65]. The more physical string-localized Hilbert space formulation is sufficiently different in order to consider this problem as open before it is also confirmed in that setting; theoretical problems cannot be solved by experimental findings.

Another limitation of the BRST formalism is that the descend to the physical Hilbert space requires an s-operation which only changes the free massive vector potential in a linear way; a nonlinear connection suggested by gauge theory as in (15) would clash with the cohomological descent from Krein to Hilbert space. The Krein method did however confirm the veracity of Stora's statement that the Lagrangian structure of gauge theory is not the result of a group theoretic imposition of symmetries, but rather a consequence of the more subtle renormalization requirement [65] in the presence of massive vectormesons.

Stringlike localization also entered the axiomatic approach to theories with mass-gaps as the *most general localization of charge-carrying fields associated with pointlike generated observables which can be derived from the mass-gap assumption*; this is the result of a deep structural theorem by Fredenhagen and Buchholz [3]. It seems likely that the strings of matter fields in massive gauge theories (which unlike the vectormeson strings cannot be removed by passing to field strength by differentiation) are generators of Buchholz-Fredenhagen spacelike-cone-localized operators. In this case the massive higher spin strings could be understood as replacing the too singular nonrenormalizable pointlike fields which may continue to exist as SLF fields in the Borchersclass of string-localize fields; which in the algebraic B-F setting indicate their presence in terms of spacelike cone localization. This may be seen as a concrete explanation as to why noncompact localization occurs even in theories with mass gaps.

3.2 Remarks on modular localization

There remains the problem of what this significant enlargement of renormalizability and localization means in terms of its physical consequences. We will return to this problem in the next subsection, after explaining some ideas about modular localization in the simplest context of Wigner representations and their relation to the operator-algebraic formulation of modular localization.

It has been realized, first in a special context in [69], and afterwards in a general rigorous setting in [19] (see also [67][20]), that there exists a *natural localization structure* on the Wigner representation space for *any* positive energy representation of the proper

[26] Exponentials of free scalar fields are SLF [62] and it is believed that this is true in general for exponentials of fields with scale dimension d=1.

Poincaré group. A convenient presentation can be given in the context of a spinless particle for which the $(m > 0, s = 0)$ Wigner one-particle space is the Hilbert space H_1 of (momentum space) wave functions with the inner product

$$H_1 : (\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{-ipx}\varphi(p)\frac{d^3p}{2p_0} \tag{18}$$

$$g \in \mathcal{P}_+ = \mathcal{P}_+^\uparrow \cup \mathcal{P}_+^\downarrow \quad U(g)(H_1^{(1)} \oplus H_1^{(2)}) = \begin{cases} U(g)H_1^{(1)}, \ g \in \mathcal{P}_+^\uparrow \\ U^{anti}(g)H_1^{(1)}, \ g \in \mathcal{P}_+^\downarrow \end{cases}$$

In this case the covariant x-space amplitude is simply the on-shell Fourier transform of this wave function, whereas for $(m \geq 0; s \geq 1/2)$ the covariant spacetime wave function is more involved as a consequence of the presence of intertwiners $u(p, s)$ between the manifestly unitary and the covariant form of the representation [56]. The second line expresses the action of the proper part of the Poincaré group $\mathcal{P}_+$ which includes all $det(g) = 1$ transformations; it consists of the action of the connected part on the irreducible Wigner representation space H_1 and the action of a time-reversing antiunitary action on a second copy of H_1 (whose wave functions refer to antiparticles which reduce to particles in the charge-neutral case).

Selecting a wedge region e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$, one notices that the unitary wedge-preserving boost $U(\Lambda_W(\chi = -2\pi t)) =: \Delta^{it}$ commutes with the antiunitary reflection J_W on the edge of the e.g. t-z wedge[27] $(x_0 \to -x_0, z \to -z;\ \vec{x}_{transvere}\ fixed)$. This has the unusual (and perhaps even unexpected) consequence that the unbounded and antilinear operator

$$\begin{aligned} S_W &:= J_W \Delta^{\frac{1}{2}}, \quad S_W^2 \subset 1 \\ since \quad & J\Delta^{\frac{1}{2}}J = \Delta^{-\frac{1}{2}} \end{aligned} \tag{19}$$

which is intrinsically defined in terms of Wigner representation data, is *involutive on its dense domain* and therefore has a unique closure with $ranS = domS$ (unchanged notation for the closure).

The involutivity means that the S-operator has ± 1 eigenspaces; since it is antilinear, the +space multiplied with i changes the sign and becomes the -space; hence it suffices to introduce a notation for just one real eigenspace

$$\begin{aligned} & K(W) = \{domain\ of\ \Delta_W^{\frac{1}{2}},\ S_W\psi = \psi\} \\ & J_W K(W) = K(W') = K(W)',\ duality \\ & \overline{K(W) + iK(W)} = H_1,\ K(W) \cap iK(W) = 0 \end{aligned} \tag{20}$$

It is important to be aware that one is dealing here with *real* (closed) subspaces K of the complex one-particle Wigner representation space H_1. An alternative is to directly work with the complex dense subspaces $K(W) + iK(W)$ as in the third line. Introducing the *graph norm* in terms of the positive operator Δ, the dense complex subspace becomes a Hilbert space $H_{1,\Delta}$ in its own right. The upper dash on regions denotes the causal

[27] Wedges in general position are obtained from the t-z wedge by Poincaré transformations.

disjoint (the opposite wedge), whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on H. All the definition work for arbitrary positive energy representations of the Poincaré group [19].

The two properties in the third line are the defining relations of what is called the *standardness property* of a real subspace[28]; any abstract standard subspace K of an arbitrary real Hilbert with a K-operator space permits to define an abstract S-operator in its complexified Hilbert space

$$\begin{aligned} &S(\psi + i\varphi) = \psi - i\varphi, \ S = J\Delta^{\frac{1}{2}} \\ &domS = dom\Delta^{\frac{1}{2}} = K + iK \end{aligned} \tag{21}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group Δ^{it} and an antiunitary reflection which generally have however no geometric interpretation in terms of localization. The domain of the Tomita S-operator is the same as the domain of $\Delta^{\frac{1}{2}}$, namely the real sum of the K space and its imaginary multiple. Note that for the physical case at hand, this domain is intrinsically determined solely in terms of the Wigner group representation theory, showing the close relation between localization and covariance.

The K-spaces are the real parts of these complex $domS$, and in contrast to the complex domain spaces they are closed as real subspaces of the Hilbert space (corresponding to the one-particle projection of the real subspaces generated by Hermitian Segal field operators). Their symplectic complement can be written in terms of the action of the J operator and leads to the K-space of the causal disjoint wedge W' (Haag duality)

$$K_W' := \{\chi | \ Im(\chi, \varphi) = 0, all \ \varphi \in K_W\} = J_W K_W = K_{W'} \tag{22}$$

The extension of W-localization to arbitrary spacetime regions $\mathcal{O}$ is done by representing the causal closure $\mathcal{O}''$ as an intersection of wedges and defining $K_{\mathcal{O}}$ as the corresponding intersection of wedge spaces

$$K_{\mathcal{O}} = K_{\mathcal{O}''} \equiv \bigcap_{W \supset \mathcal{O}''} K_W, \quad \mathcal{O}'' = causal\ completion\ of\ \mathcal{O} \tag{23}$$

These K-spaces lead via (21) and (23) to the modular operators associated with $K_{\mathcal{O}}$.

For those who are familiar with Weinberg's intertwiner formalism [56] relating the (m,s) Wigner representation to the dotted/undotted spinor formalism, it may be helpful to recall the resulting "master formula"

[28]According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

$$\Psi^{(A,\dot{B})}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int (e^{-ipx} \sum_{s_3=\pm s} u^{(A,\dot{B})}(p,s_3) a(p,s_3) + \tag{24}$$
$$+ e^{ipx} \sum_{s_3=\pm s} v^{(A,\dot{B})}(p,s_3) b^*(p,s_3)) \frac{d^3p}{2\omega}$$
$$\sum_{s_3=\pm s} u^{(A,\dot{B})}(p,s_3) a(p,s_3) \to u(p,e) \cdot a(p)$$

where the a, b amplitudes correspond to the Wigner momentum space wave functions of particles/antiparticles and the u, v represent the intertwiner and its charge conjugate. For the third class (infinite spin, last line), the sum over spin components has to be replaced by an inner product between a p, e-dependent infinite component intertwiner u and an infinite component $a(p)$ since in this case Wigner's "little space" is infinite dimensional. The $\Psi(x)$ and $\Psi(x,e)$ are "generating wave functions" i.e. they are wavefunction-valued Schwartz distributions which by smearing with $\mathcal{O}$-supported test functions become $\mathcal{O}$-localized wave functions. Adding the opposite frequency antiparticle contribution, one obtains the above formula which, by re-interpreting the $a^{\#}, b^{\#}$ as creation/annihilation operators (second quantization functor) and the $u(p)$ by $u(p,e)$, describes point- or string-like free fields. The resulting operator-valued Schwartz distributions are "global" generators in the sense that they generate $\mathcal{O}$-localized operators $\Psi(f)$ for *all* $\mathcal{O}$ by "smearing" them with $\mathcal{O}$-supported test functions $suppf \in \mathcal{O}$.

Only in the massive case the full spectrum of spinorial indices $A, \dot{B}$ is exhausted (9), whereas the massless case leads to restrictions (10) which come about because pointlike "field-strength" are allowed, whereas pointlike "potentials" are rejected (related to the different zero mass little group). This awareness about the conceptual clash between localization and the Hilbert space[29] is important for the introduction of string-localization.

Whereas Weinberg [56] uses (the computational somewhat easier manageable) covariance requirement[30], the modular localization method is based on the direct construction of localized Wigner subspaces and their stringlike generators. In that case the intertwiners depend on the spacelike direction e which is not a parameter but, similar to the localization point, a variable in terms of which the field fluctuates [20] and whose presence allows the short distance fluctuations in x to be more mild than in case of their pointlike counterparts.

The the short-distance improving property of the generating stringlike fields is indispensable in the implementation of renormalizable perturbation theory in Hilbert space for interactions involving spins $s > 1/2$[31]. Whereas pointlike fields are the mediators between classical and quantum localization, the stringlike fields are outside the Lagrangian or functional quantization setting since they are not solutions of Euler-Lagrange equations; as already Weinberg noticed one does not need a free Lagrangion in order to writen

[29] In the case of [20] this awareness came from the prior use of "modular localization" starting in [68][69] but foremost (covering *all* positive energy Wigner representations) in [19].

[30] For wave functions and free fields covariance is synonymous with causal localization, but in the presence of interaction the localization of operators and that of states split apart.

[31] These are also prcisely those interactions in which the absence of mass gaps does not lead to problems with the particle structure.

down Feynman rules for arbitrary spin. This fact is important in causal perturbation theory where any scalar Wick polynomial of spinorial fields can be used whereas Lagrangian and functional quantization needs the full action. String-localization lowers the power-counting limit, but renders the application of the iterative Epstein-Glaser machinery [12] more involved. In the next section it will be shown that modular localization is essential for *generalizing Wigner's intrinsic representation theoretical approach to the realm of interacting observable algebras.*

In order to arrive at Haag's algebraic setting of local quantum physics in the absence of interactions, one may avoid "field coordinatizations" and apply the Weyl functor Γ (or its fermionic counterpart) directly to *wave function subspaces* where upon they are functorially passed directly to operator algebras, symbolically indicated by the functorial relation

$$K_{\mathcal{O}} \overset{\Gamma}{\to} \mathcal{A}(\mathcal{O}) \tag{25}$$

The functorial map Γ *also* relates the modular operators S, J, Δ from the Wigner setting with their "second quantized" counterparts $S_{Fock}, J_{Fock}, \Delta_{Fock}$ in Wigner-Fock space; it is then straightforward to check that they are precisely the modular operators of the Tomita-Takesaki modular theory applied to causally localized operator algebras (using from now on the shorter S, J, Δ notation also for modular objects in operator algebras).

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O})) &\equiv \Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it} = \mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J &= \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned} \tag{26}$$

In the absence of interactions these operator relation are consequences of the modular relations for Wigner representations. The *Tomita-Takesaki theory secures their general existence for standard pairs* $(\mathcal{A},\Omega)$ i.e. an operator algebras $\mathcal{A}$ and a state vector $\Omega \in H$ on which $\mathcal{A}$ acts cyclic and separating (no annihilators of Ω in $\mathcal{A}$). The polar decomposition of the antilinear closed Tomita S-operator leads to the unitary modular automorphism group Δ^{it} associated with the subalgebra $\mathcal{A}(\mathcal{O}) \subset B(H)$ and the vacuum state vector Ω i.e. with the pair $(\mathcal{A}(\mathcal{O}),\Omega)$.

Although $B(H)$ is generated from the two commuting algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O})'$, they do not form a tensor product in $B(H)$; hence the standard quantum-information and QM concepts concerning entanglement and density matrices are somewhat different; the QFT realization of entanglement for monads is stronger, it leads to a more singular form of entanglement in which impure states cannot be described in terms of density matrices [32]. As a result of this "monad-type entanglement" the impure state results from just *restricting the global vacuum to the local monad*, one does not have to average over degrees of freedom in order to convert entangled states into density matrices (as it would be necessary in the standard quantum information situation where instead of a monad one has a $B(H)$ type algebra associated with a factor space H).

As mentioned, modular localization of operators is more restrictive than modular localization of states. It is perfectly conceivable that a state vector generated by applying an *algebraically indecomposable* stringlike localized field to the vacuum is decomposable

[32] The localization entropy of the vacuum entanglement for $\mathcal{A}(\mathcal{O})/\mathcal{A}(\mathcal{O}\prime)$ is infinite.

into a direct sum/integral over pointlike generated Wigner representations; in fact all positive energy representations which do not contain components to the infinite spin representations allow such generally continuous decomposition. An important illustration of this fact are the charge-carrying infraparticle fields in QED.

The only case for which the modular localization theory (the adaptation of the Tomita-Takesaki modular theory to the causal localization principle of QFT) has a geometric interpretation independent of whether interactions are present or not and independent of the type of quantum matter, is the wedge region i.e. the Poincaré transforms of the standard wedge $W = \{x_0 < x_3 | \mathbf{x}_{tr} \in \mathbb{R}^2\}$. In that case the modular group is the wedge-preserving Lorentz boost, and the J represents a reflection on the edge of the wedge i.e. it is up to a π-rotation equal to the antiunitary TCP operator. The derivation of the TCP invariance as derived by Jost [42], together with scattering theory (the TCP transformation of the S-matrix) leads to the relation

$$J = S_{scat} J_{in} \tag{27}$$

which in [68][69] has been applied to constructive problems of integrable QFTs. This is a relation which goes much beyond scattering theory; in fact it only holds in *local* quantum physics where it attributes the new role of a relative modular invariant of causal localization to the S-matrix which S_{scat} does not have in QM.

This opens an unexpected possibility of a new access to QFT, in which the first step is the construction of generators for the wedge-localized algebra $\mathcal{A}(W)$ with the aim to obtain spacelike cone-localized (with strings as a core) or double cone-localized algebras (with a point as core) from *intersecting wedge algebras.* In this top-to-bottom approach, which is based on the intuitive idea that the larger the localization region the better the chance to construct generators with milder vacuum polarization, pointlike fields would only appear at the end of the construction. In fact according to the underlying philosophy that all relevant physical data can be obtained from localized algebras, the use of individual operators (apart from the distinguished conserved currents of inner symmetries) within such an algebra may be avoided althogether; the *relative positioning of the localized algebras (monads) should account for all physical phenomena in particle physics*, monads by themselves have no individuality, they are all isomorphic The next section presents the first step in such a construction.

The only prerequisites for the general (abstract) case is the "standardness" of the pair $(\mathcal{A}, \Omega)$ where "standard" in the theory of operator algebras means that Ω is a cyclic and separating vector with respect to $\mathcal{A}$, a property *which in QFT is always fulfilled* for localized $\mathcal{A}(\mathcal{O})'s$, thanks to the validity of the Reeh-Schlieder theorem [3]. These local operator algebras of QFT are what has been referred as *monads* in previous publications [10]*;* as mentioned before, they are remarkably different from the algebra of all bounded operators $B(H)$ which one encounters for Born-localized algebras in QM [11]. For general localization regions the one-parametric modular unitaries have no geometric interpretation (they describe a kind of fuzzy action inside $\mathcal{O}$), but they are uniquely determined in terms of intersections of their geometric W-counterparts and are expected to become important in any top-to-bottom construction of models of QFT. Even in the simpler context of localized subspaces $K_{\mathcal{O}}$ related to Wigner's positive energy representation theory for the

Poincaré group and its functorial relation to free fields, these concepts have shown to be useful [19].

The most important conceptual contribution of modular localization theory in the context of the present work is the assertion that the reduction of the global vacuum (also finite energy particle states) to a local operator algebra $\mathcal{A}(\mathcal{O})$ leads to temperature-like states for which the "thermal Hamiltonian" H_{mod} is the generator of the modular unitary group

$$e^{-i\tau H_{mod}} := \Delta^{i\tau} \qquad (28)$$
$$\langle AB \rangle = \left\langle Be^{-H_{mod}}A \right\rangle$$

where the second line has the form is what one obtains for heat bath thermal systems after rewriting the Gibbs trace formula into the state-setting of the *open system formulation* of statistical mechanics[33] [3]. Whereas the trace formulation breaks down in the thermodynamic limit, this analytic KMS formula (asserting analyticity in $-1 < Im\tau < 0$) remains. It is in this and only in this thermodynamic limit, that a monad algebra appears also in QM.

As mentioned in the introduction, the intrinsic thermal aspect of localization is the reason why the probability issue in QFT is conceptually radically different from QM for which one has to add the Born probability; localization and probability are added to QM since as a global theory these aspects are not intrinsic.

Closely related to a modular localization is the "GPS characterization" of a QFT (including its Poincaré spacetime symmetry, as well as the internal symmetries of its quantum matter content) in terms of modular positioning of a finite number of monads in a shared Hilbert space. For d=1+1 chiral models the minimal number of copies is 2, whereas in d=1+3 the smallest number for a GPS construction is 7 [70]. This way of looking at QFT is an *extreme relational point of view* in terms of objects which have no internal structure by themselves; this explains the terminology "monad" (Leibnitz's point of view about reality but now in the context of quantum matter) [70][11]. This view of QFT exposes its radically holistic structure in the most forceful way. In praxis one starts with one monad and assumes that one knows the action of the Poincaré group on it [68][69]. This generates a net of transformed monads which by forming intersections lead to monads associated to smaller regions (spacelike cones or double cones). This was precisely the way in which the existence of factorizing models was shown [23], where the nontriviality of the intersection was established by verifying the "modular nuclearity property" of degrees of freedom.

In order to show the power of this new viewpoint for particle physics, the following last subsection shows some different viewpoints about some open problems in Standard Model physics.

3.3 Expected consequences for Standard Model physics

Since its inception the "Higgs mechanism" has been the cause of many conceptual misunderstandings [71]. There were two interpretations namely as a *spontaneous symmetry*

[33] Ground state problems in QM do not come anywhere near such a tight situation.

breaking in which the zero mass Goldstone degree of freedom was used to convert a photon with two helicity degrees of freedom into a massive vectormeson with 3 polarization degrees of freedom, or as a kind of *screened electromagnetism.* The screening idea can be traced back to Schwinger [72] who thought that QED may exist in another phase in which the Maxwell charge is screened (i.e. the integral over the zero component of the Maxwell current vanishes) and the massless vectorpotential becomes massive. In this case the Maxwell charge vanishes, whereas the spontaneous symmetry breaking mechanism leads to an infinite charge as the result of the coupling of a the current to a zero mass Goldstone boson

$$\begin{aligned} screening: \ Q = \int j_0(x)d^3x = 0, \ \partial^\mu j_\mu = 0 \qquad (29) \\ spont.symm. - breaking: \int j_0(x)d^3x = \infty \end{aligned}$$

Obviously the analogy to the quantum mechanical Debeye screening led him this idea. In that case long range potentials between electrically charged particles in a medium in which both $\pm$ charges are present become effectively short ranged. But whereas in QM this is an *effective* mechanism which does not alter the fundamental quantum mechanical structure, screening in QFT is more radical. The conceptually different structure of causal QED requires *a change of particle spectrum from photons to massive vectormesons* and a change of the charge matter from complex to real; hence such analogies have to be taken with a grain of salt. But it has a certain intuitive appeal to picture the original model of a real scalar field interacting with a massive vectorpotential as proposed by Higgs as the screened form of scalar QED in which the imaginary component of the scalar charged field converts the two helicity component photon into the three spin component vectormeson (the Proca field).

Remembering that scalar QED is a renormalizable theory with two couplings, the presentation of the Higgs model as *screened scalar QED* seems to be conceptually better justified than assigning to a Goldstone boson the role of "fattening" the photon, although a pragmatist may maintain that metaphoric arguments are acceptable as long as the result is intrinsically consistent. But even the staunchest pragmatist probably would have changed his mind, after becoming aware of the following theorem

Theorem 2 *(Swieca [73]) In abelian gauge theories with a mass gap the Maxwell charge is screened*

$$mass\ gap\ and\ \partial^\nu F_{\mu\nu} = j_\mu \curvearrowright Q = \text{"} \int d^3x j_0(x) \text{"} = 0 \qquad (30)$$

This theorem was later generalized in [51]. It does not distinguish between elementary mass and a mass from the Higgs mechanism. But this is because also experiments cannot distinguish between these possibilities. The Higgs mechanis was the historical path by which physicists familiar with Goldstones theorem discovered couplings between massive vectorpotentials and scalar neutral ("screened") came to light, but now that we know that the possibility of such renormalizable couplings has been simply overlooked before

Higgs we can dispense such "pons asini" which have no counterpart in nature. The only theorems which prevent conserved charges to lead to nontrivial charges are the Goldstone theorem for spontaneous broken symmetries and Swiecas screening theorem which attributes mathematical meaning to Schwinger's screening ideas. The question of whether a model which couples massive vectormesons also needs a coupling to scalar particles for reasons of consistency is an issue which has to be settled by studing its renormalization theory; for massive spinor and scalar QED the answer is negative and for Y-M couplings the situation is as yet undecided (see later).

It is somewhat surprising that the much more physical screening picture for massive vectormesons did not take hold in the 70s [71]. A possible explanation is perhaps that the incorrect idea that massive vectormesons are only renormalizable if they receive their mass through a Higgs mechanism in the sense of a broken symmetry took hold more rapidly than more critical ideas. From a modern point of view *the Higgs model is the nothing else than a renormalizable model which couples a massive vectormeson to massive scalar real field* [34]; metaphoric pictures as spontaneous symmetry breakings only detract, and even the correct Schwinger-Swieca screening in terms of a screened Maxwell charge is a pons asini albeit one which is not in contradiction with known facts.

The idea of a scalar particle like the Dear Lord gives "massive life" to poor massless creatures (presumably including the scalar itself) i.e. the metaphoric fattening idea from spontaneous symmetry breaking does not become less metaphoric when CERN uses it to sell its experimental result. The necessity to sell expensive experiments to the public by using God's name could be damaging to particle physics. It puts unnecessary strain on experimentalists to verify theoretical ideas for which for more than 4 decades no alternative more foundational idea was developed. It is an interesting question whether this stagnation could not have been avoided if the sociological success of ST did not make particle physics susceptible to metaphoric presentations.

As mentioned, in the BRST treatment of massive selfcoupled Y-M vectormesons one needs such a scalar coupling [65], but as a result of the limitations of this method one has to wait for a confirmation from the adiabatic equivalence formulation in the stringlocal Hilbert space formulation based on physical principles rather than prescriptions. With or without such a real (Higgs) field, in any case the Maxwell current (the current which appears on the right hand side of the divergence of F) according to the previous theorem is always screened and the only kind of symmetry breaking is the absence of Z_2 symmetry as a result of the appearance of odd terms in the consistent couplings of massive vectorpotentials with real scalar field (completely determind by renormalizability [65]). This is in complete agreement with recent findings in [74] since in all massive vectormeson models, no matter by which method they have been obtained, it is a fact that the identically conserved Maxwell current which appears on the right hand side of the divergence of the curl of the massive vectorpotentials always leads to a screened charge. Other conserved currents in the theory are of course not screened. From the viewpoint of the adiabatic equivalence in the Hilbert space, the necessity of an additional scale field for obtaining consistent renormalizable massive Y-M couplings does not look very plausible. But this problem should be carefully scrutinized, since an implausible appearance is not

[34] All strictly renormalizable models are uniquely determined by their field content; the concrete form follows from renormalization [65].

an argument in such a conceptually subtle region of QFT.

A second issue which is not yet on safe grounds in the sense mathematical physics, is the confirmation of asymptotic freedom through the property of the beta-function which appears in the parametric Callen-Symanzik equation for correlation functions. The existing calculation methods first establish the C-S equations in massive theories and then argue that since the beta function is independent of masses, the same beta must also be valid in the zero mass limit if it exists. This suggests to derive the C-S equation in the string-localized formulation which at the time of writing of this paper has not been done yet. It may be easier to do this in the more familiar Krein formulation since many second order computations have already been done in that setting. Even though that pointlike formulation has no massless limit, beta functions in parametric C-S differential equations are interesting in their own right, simply because they have not been done.

Perturbative beta functions from analytic continuation in the spacetime dimension without correlations and C-S equations by claiming that the theory has no perturbative prescription for long distances[35] simply lacks credibility especially when one already knows that a zero mass pointlike description does not exist. Infrared-finite string-localized prescriptions are expected to be behind the on-shell infrared divergecies as they occur e.g. in QED. Whether they also bear on confinement problems is not known. Since the divergence of perturbative series (including string localization) has no bearing on the nonperturbative existence and its properties, the infrared convergence of off-shell perturbation theory has no direct information about the nature of confinement.

In stringlike cases the mathematical description in terms of off-shell correlation functions may *not be enough for the physical interpretation.* Even in a string-localized formulation of QED with off-shell correlations of charge matter fields, one lacks a spacetime idea how to relate the string-localized "infraparticle" fields with particle-like on-shell objects and their scattering processes. The description of scattering of electrically charged particle in terms of recipe for photon-inclusive cross sections which starts with the Bloch-Nordsiek model reached a form of useful covariant recipes for given photon resolution in the Yenni-Frautschi-Suura formalism. But compared with the spacetime-based LSZ reduction formalism these successful descriptions always appeared somewhat contrived. Ideas to derive them from spacetime localization principles based on the Huygens property go back to early work by Buchholz [3] and they were recently taken up again. There is a good chance that they may bridge the conceptual abyss between the well-understood scattering formalism for situations with mass gaps and the conceptually much lesser understood prescriptions in the case of presence of massless vector potentials. In [37] Buchholz reports on an impressive new attempt which is based on a distinction between superselection sector and charge classes and which "tames" the unwieldy soft photon clouds. They are removed by restricting states to the (any) forward light cone. The so-restricted equivalence classes of states describe *charge classes* which cut-off the infinite infrared clouds in an observer-independent intrinsic way; through them the infraparticle aspect disappears and gives way to particles with sharp masses without ever to have to introduce ad hoc infrared cutoffs and photon resolutions. The analysis of charge classes leads to compact symmetry groups in analogy to the case with mass gaps. But the study of this connection between

[35] The correct statement would be that it has off-shell infrared divergencies in the (nonexisting) pointlike description.

the string-localized perturbation theory and string-free scattering for charged classes is still a problem of future research. In any case the connection between string-localized massive theories and their massless limit has no on-shell counterpart.

The reader may wonder why the word "supersymmetry" occurred in this paper only in connection with the solution of the mathematical Majorana problem (the infinite component "superstring representations" of the Poincaré group) and not with particle physics. There is a simple answer; whereas the main physical motivation for supersymmetry, namely the improvement of short distance properties in order to increase the range of renormalizability, turned out to be an illusion, the use of string-localized fields in Hilbert space really adds to the finite number of pointlike renormalizable couplings infinitely many renormalizable stringlike interactions for higher spin field. Of course not all of them possess compactly localized observable subalgebras which seems to be the prerequisite for their physical acceptance.

4 Generators of wedge algebras, extension of Wigner representation theory in the presence of interactions

Theoretical physics is one of the few areas of human endeavor in which the identification of an error may be as important as the discovery of a new theory. This is especially the case if the committed error is related to a lack of understanding or misunderstanding of a central principle as causal localization. Whereas off-shell analytic properties of correlation function were systematically analyzed in the pathbreaking work of Bargmann, Hall and Wightman, it was already clear at the time of the dispersion relations that on-shell analytic properties are of a different conceptual caliber and that the field-particle relation coming from LSZ scattering theory is not sufficient for for their understanding. In some special cases of elastic scattering the application of the intricate mathematics of several complex variables and the formation of natural analytic extensions [14] led to a proof of the crossing analyticity. But the derivation did not reveal much about what we know nowadays, namely that particle crossing identity is closely related (and in fact can be derived from) to the KMS identity of modular wedge localization. The main difference to the Unruh effect is that that one has to convert field states in the presence of interactions into particle states; but this again can be achieved in terms of modular localization.

Only after the arrival of modular localization and its role in the construction of d=1+1 integrable models for the spacetime identification of the Zamolodchikov algebra structure [68][69], the understanding of these properties began to improve. The crucial step was the realization that the S-matrix was not only an operator resulting from time-dependent scattering theory (which it is in every QT), but also a relative modular invariant of wedge-localized algebras. This led to the idea that the crossing property and its analytic aspects in terms of particle rapidities are a result of a particle translation of the analytic KMS identity for operators localized in the wedge, for which the analyticity refers to the hyperbolic angle of the wedge-preserving Lorentz transformation. The derivation of the crossing relation from the same modular localization principle which solves the E-J conundrum and explains the Unruh effect is surprising; this and some remarks on a closely related proposal for a general on-shell construction [10] (which extends the successful construction of integrable models from the structure of their generators of wedge algebras [23]) is the topic of this section.

In this way *the original aim* of Mandelstam's on-shell project for finding a route to particle theory, which is different to quantization and perturbation theory and stays closer to directly observational accessible objects, is recovered, and the picture puzzle trap of ST (section 2) which led to wrong understandings of crossing is avoided. It is closely related to the top-to-bottom oriented LQP setting in which the desired concepts are laid down before their mathematical and computational implementation starts; this is opposite to quantization approaches were the properties of objects and their interpretation come to light only after having done the calculations. Besides aspects which are accessible by quantization, there are also properties which cannot be understood in this way, as the E-J conundrum or other thermal[36] aspects of modular localization as the Unruh and Hawking effects, as well as localization-caused entropy which characterize the modular statistical mechanics aspect of localized ensembles. This section adds the particle crossing and the closely related on-shell construction method to these properties whose understanding requires the use of the modular localization principle.

In retrospect it is clear why Mandelstam's project had no chance to succeed in the 60s and 70s; the necessary conceptual tools were not available at a time in which the impressive success of renormalized perturbation was still very much on peoples mind, and QFT was considered simply as that theory behind Lagrangian/functional quantization.

The most difficult aspect of modular localization is the comprehension of the big separation it creates between particles and fields in the presence of interactions. Whereas these two concepts are related in a functorial way in the absence of interactions, the presence of interactions separates them in such a way that it takes great conceptual efforts to understand what kind of connection is left. This effort goes significantly beyond the use of modular localization needed for the E-J conundrum and Unruh-Hawking effects. It starts with observation that the S-matrix is not only that well-known object resulting from the well-understood relation between the large time asymptotic behavior of fields with particles (which it is in any QT). In modular localizable theories as QFT, it is also a relative modular invariant associated with the structure of an interacting wedge algebra relative to its free counterpart (generated by incoming fields) which represents the needle's pin through which particles become related with interacting fields.

In order to motivate the reader to enter a journey which takes him far away from textbook QFT, it is helpful to start with a theorem which shows that the familiar particle-field relations breaks down in the presence of *any* interaction. The following theorem shows that the separation between particles and *interacting* localized fields and their algebras is very drastic indeed [10]:

Theorem 3 *(Mund's algebraic extension [75] of the old J-S theorem [27]) A Poincaré-covariant QFT in $d \geq 1+2$ fulfilling the mass-gap hypothesis and containing a sufficiently*

[36] Here thermal is not necessarily referring to what can be measured with a thermometer [6] but rather characterizes the specific KMS (modular) impurity which results from a $\mathcal{A}(\mathcal{O})$-restriced vacuum.

large set of "temperate" wedge-like localized vacuum polarization-free one-particle generators (PFGs) is unitarily equivalent to a free field theory.

It will be shown in the following that the requirement of temperateness of generators (Schwartz distributions, equivalent to the existence of a translation covariant domain for PFG's [76]) is very strong, it only allows integrable models and integrability in QFT can only be realized in d=1+1. Note that Wightman fields are assumed to be operator-valued temperate distributions. Hence the theorem says that even in case of a weak localization requirement (such as wedge-localization), one cannot find interacting PFGs with translation invariant domain properties. However any QFT permits wedge-localized nontemperate generators [76]. The theorem has a rich history which dates back to Furry and Oppenheimer's observation (shortly after Heisenberg's discovery of localization-cause vacuum polarization) that Lagrangian interactions always lead to fields which, if applied to the vacuum, inevitably create a particle-antiparticle polarization cloud in addition to the desired one-particle state.

The only remaining possibility to maintain a relation between a **p**olarization-**f**ree **g**enerator (PFG) leading to a pure one-particle state and a localized operator (representing the field side) has to go through the bottleneck of nontemperate PFG generators of wedge-localized algebras; this is all which remains of the functorial particle-field relation between Wigner particles and fields. It makes the extension of the Wigner idea of using representation theory in the presencence of interactions (the on-shell construction project) very subtle since one has to use wedge-localized multiparticle states from the beginning.

The idea is to construct a kind of *"emulation" of wedge-localized free incoming fields* (˜particles) *inside the interacting wedge algebra* as a replacement for the nonexisting second quantization functor for Wigner particles. As the construction of one-particle PFGs, this is achieved with the help of modular localization theory.

The starting point is a *bijection* between wedge-localized incoming fields operators and interacting operators [47][10]. This bijection is based on the equality of the dense subspace which these operators from the two different algebras create from the vacuum. Since the domain of the Tomita S operators for two algebras which share the same modular unitary Δ^{it} is the same, a vector $\eta \in domS \equiv domS_{\mathcal{A}(W)} = dom\Delta^{\frac{1}{2}}$ is also in $domS_{\mathcal{A}_{in}(W)} = \Delta^{\frac{1}{2}}$ (in [76] it was used for one-particle states). In more explicit notation, which emphasizes the bijective nature, one has

$$\begin{gathered} A\left|0\right\rangle = A_{\mathcal{A}(W)}\left|0\right\rangle, \ A \in \mathcal{A}_{in}(W), \ A_{\mathcal{A}(W)} \in \mathcal{A}(W) \\ S(A)_{\mathcal{A}(W)}\left|0\right\rangle = (A_{\mathcal{A}(W)})^{*}\left|0\right\rangle = S_{scat}A^{*}S_{scat}^{-1}\left|0\right\rangle, \ S = S_{scat}S_{in} \\ S_{scat}A^{*}S_{scat}^{-1} \in \mathcal{A}_{out}(W) \end{gathered} \tag{31}$$

Here A is either an operator from the wedge localized free field operator algebra $\mathcal{A}_{in}(W)$ or an (unbounded) operator affiliated with this algebra (e.g. products of incoming free fields $A(f)$ smeared with f, $suppf \in W$); S denotes the Tomita operator of the interacting algebra $\mathcal{A}(W)$. Under the assumption that the dense set generated by the dual wedge algebra $\mathcal{A}(W)'\left|0\right\rangle$ is in the domain of definition of the bijective defined "emulats" (of the wedge-localized free field operators inside the interacting counterpart) the $A_{\mathcal{A}(W)}$ are uniquely defined; in order to be able to use them for the reconstruction of $\mathcal{A}(W)$ the domain should be *a core for the emulats.* Unlike smeared Wightman fields, the emulats $A_{\mathcal{A}(W)}$ do not define a polynomial algebra, since their unique existence does not allow to impose additional domain properties as successive applicability; in fact they only form a vector space, and the associated algebras have to be constructed by spectral theory or other means for the extraction of an algebra from a vector space of closed operators.

Having settled the problem of uniqueness, the remaining task is to determine their action on wedge-localized multi-particle vectors and to obtain explicit formulas for their particle formfactors. All these problems have been solved in case the domains of emulats are invariant under translations; in that case the emulats possess a Fourier transform [76]. This requirement is extremely restrictive and is only compatible with d=1+1 elastic two-particle scattering matrices of integrable models[37]; in fact it should be considered as the *foundational definition of integrability of QFT in terms of properties of wedge-localized generator* [10].

Since the action of emulats on particle states is quite complicated, we will return to this problem after introducing some useful notation.

For integrable models the wedge duality requirement (**??**) leads to a unique solution (the Zamolodchikov-Faddeev algebra), whereas for the general non-integrable case we will present arguments, which together with the comparison with integrable case determine the action of emulates on particle states. The main additional assumption is that the only way in which the interaction enters this construction of bijections is through the S-matrix[38] which amounts to saying that a LQP (but not its coordinatization in terms of fields) is uniquely determined in terms of the S-matrix. With this assumption the form of the action of the operators $A_{\mathcal{A}(W)}$ on multiparticle states is fixed. The ultimate check of its correctness through the verification of wedge duality (**??**) is a difficult problem left to future investigations.

Whereas domains of emulats in the integrable case are translation-invariant [76], the only domain property which is *always* preserved in the general case is the invariance of the domain under the subgroup of those Poincaré transformations which leave W invariant. In contrast to QM, for which integrability occurs in any dimension, integrability in QFT turns out to be restricted to d=1+1 factorizing models [76][10].

A basic fact in the derivation of the crossing identity, including its analytic properties which are necessary in order to return to the physical boundary, is the *cyclic KMS property.* For three operators affiliated with the interacting algebra $\mathcal{A}(W)$ (two of them being emulates of incoming operators[39]) it reads:

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[37]This statement, which I owe to Michael Karowski, is slightly stronger than that in [76] in that that higher elastic amplitudes are combinatorial products of two-particle scattering functions, i.e. the only solutions are the factorizing models.

[38]A not unreasonable assumption because this is the only interaction-dependent object which enters as a relative modular invariant the modular theory for wedge localization.

[39]There exists also a "free" KMS identity in which B is replaced by $(B)_{\mathcal{A}_{in}(W)}$ so everything refers to the algebra $\mathcal{A}_{in}(W)$. The derivation of the corresponding crossing identity is rather simple [10] and closely related to the Wick-ordering formalism..

$$\left\langle 0|BA^{(1)}_{\mathcal{A}(W)}A^{(2)}_{\mathcal{A}(W)}|0\right\rangle \overset{KMS(\mathcal{A}(W))}{=} \left\langle 0|A^{(2)}_{\mathcal{A}(W)}\Delta BA^{(1)}_{\mathcal{A}(W)}|0\right\rangle \tag{32}$$
$$A^{(1)} \equiv: A(f_1)...A(f_k):,\ A^{(2)}_{in} \equiv: A(f_{k+1})...A(f_n):,\ suppf_i \in W$$

where in the second line the operators were specialized to Wick-ordered products of smeared free fields $A(f)$ which are then emulated within $\mathcal{A}(W)$. Their use is necessary in order to convert the KMS relation for $\mathcal{A}(W)$ into an identity of *particle formfactors* of the operator $B \in \mathcal{A}(W)$. If the bijective image acts on the vacuum, the subscript $\mathcal{A}(W)$ for the emulats can be omitted and the resulting Wick-ordered product of free fields acting on the vacuum describes a multi-particle state in $\hat{f}_i$ momentum space wave functions. The roof on top of f denotes the wave function which results from the forward mass shell restriction of the Fourier transform of W-supported test function. The result are wave functions in a Hilbert space of the graph norm $\left(\hat{f}, (\Delta+1)\,\hat{f}\right)$ which forces them to be analytic in the strip $0 < Im\theta < \pi$.

The easy part in the particle transcription of the KMS relation (32) is the right hand side. Letting the hermitian conjugate of $\Delta^{\frac{1}{2}}A^{(2)}_{\mathcal{A}(W)}$ act on the bra vacuum and using its modular representation (31) one obtains an outgoing n-k state in which the particles have been changed into their antiparticles; the application of the remaining $\Delta^{\frac{1}{2}}$ amounts to an analytic continuation of the antiparticle rapidities by $i\pi$ so that the net result is the analytically continued formfactor of B between a n-k outgoing bra antiparticle state and an incoming k-particle state.

As will be seen the left hand side in (32) can, under special ordering conditions for the n rapidities, be replaced by an n-particle incoming vector which then represents the desired crossing relation. For simplicity of notation we specialize to d=1+1 in which case neither the wedge nor the mass-shell momenta have a transverse component and particles are characterized by their rapidity. Up to now the KMS relation only reads

$$\int ..\int \hat{f}_1(\theta_1)...\hat{f}_1(\theta_n)F^{(k)}(\theta_1,...,\theta_n)d\theta_1...d\theta_n = 0 \tag{33}$$
$$F^{(k)}(\theta_1,..,\theta_n) := \left\langle 0\left|BA^{(1)}_{\mathcal{A}(W)}(\theta_1,..,\theta_k)\right|\theta_{k+1},..,\theta_n\right\rangle_{in} -$$
$$- \ {}_{out}\left\langle \bar{\theta}_{k+1},..,\bar{\theta}_n\left|\Delta^{\frac{1}{2}}B\right|\theta_1,..,\theta_k\right\rangle_{in}$$

where $\bar{\theta}$ refers to antiparticle rapidities and the $\Delta^{\frac{1}{2}}$ of Δ was used to re-convert the antiparticle wave functions in the outgoing bra vector back into the original particle wave functions [10].

There are two steps which remain to be shown

1. For ordered rapidities $\theta_1 > ... > \theta_n$

$$\left\langle 0\left|BA^{(1)}_{\mathcal{A}(W)}(\theta_1,..,\theta_k)\right|\theta_{k+1},..,\theta_n\right\rangle_{in} = \left\langle 0\left|B\right|\theta_1,..,\theta_n\right\rangle_{in}$$

2. $F^{(k)}$ is locally square integrable

The first property is part of an analytic interpretation: the n-particle component of a local operator is the boundary value of a multivalued function in the multivariable θ space. One uses the statistics degeneracy of the n-particle vector to encode it into the θ-ordering; any other order correspond to another boundary value of the formfactor which results from the particular analytic continuation used to arrive at the re-ordered θ-configuration. Its physical interpretation is very different from the original n-particle interpretation in fact in general the new object represents a new state[40]. The derivation of the crossing identity does not require an operational identification of other boundary values because the ordering of the θ remains fixed (fixed L^2 wave functions with ordered support) in the derivation of the crossing identity. The only place where a physical idea enters in addition to the KMS identity is in the assumption that the singularies near the boundary are exhaused by the known multiparticle threshold cuts. Without knowing anything about the distributional nature of boundary values (in this case the local L^2 integrability) one cannot use the L^2 denseness property of wedge-localized wave functions.

For the formulation of an on-shell construction project one needs more. The only known way goes via an assumption about an operational interpretation of the analytic reordering i.e. about the operational meaning of analytic θ-reorderings in states and their possible dependence on the analytic path taken to get to the reordered configuration. This is tantamount to knowing the action of a PFG or a more general emulat on particle states beyond the vacuum. The guiding idea is that if one rapidity, say the first one in an n-particle state, is outside its ordered position then the commutaion with a k-particle cluster which is necessary to get it their only depends on this k-cluster and is described in terms of a "grazing shot" S-matrix in which there is no direct interaction within the cluster but only that part of the interaction which the θ_1 causes in order to bring it into its ordered position. The implementation of this idea requires some new concepts and necessitates abbreviated notation in order to avoid messy formulas. It did not yet path its crucial test of "wedge duality" which would show its correctness.

However for d=1+1 integrable models it undergoes a significant simplification which allows to check the wedge-duality property. In this case the on-shell generating PFGs of the interacting wedge algebra fulfill the commutation relations of the Zamolodchikov-Faddeev algebra [68][69] and can be used to construct the compact localized double cone algebra and in this way show the mathematical existence of QFTs with realistic strictly renormalizable short-distance behavior [23] (the first time in the long almost 90 year old history of QFT).

5 Resumé and concluding remarks

The main point of the present work was to explain why the important project of a mass-shell based top-to-bottom approach in particle theory took a wrong turn when, as a consequence of the insufficient understanding of the relation between on-shell analytic properties and the intrinsic localization properties of local quantum physics at that time, the dual model crossing was mistakenly accepted as describing the on-shell particle cross-

[40]For explanatory simplicits we use the terminology "state", but in reality we talk about what happens to formfactors when their particle rapidities are being analytically continued.

ing. In order to underline the subtlety of this issue it was useful to go back to the beginnings of QFT and point to a non-understood aspect of the E-J conundrum: the modular localization property as the defining property of LQP i.e. of *QFT unchained from quantization*.

Looking back at the attempts of the 60s with present hindsight, it is clear that there was not much of a chance at that time for understanding the subtle conceptual nature of analytic on-shell properties and in particular the role particle crossing property in an on-shell construction project. This also means that after having obtained *the meromorphic dual model functions* there was no chance to get out of the mentioned picture puzzle trap. Finding structures by by mathematical imagination on a subject for which the time of its physical conceptual understanding has not yet arrived is not without a risk, this is the other side of a "gift from the 21st century which fell by chance onto the 20^{th}". If one follows Feynman's "jump into the conceptual blue yonder" one better makes sure that one starts from a solid vantage point to which one can return if the jump, as it is often the case, does not lead to tangible results. In case already this vantage point is on shaky grounds, as in the case of analytic on-shell properties and ideas about particle crossing in the 60s, there is little chance of a return. This seem to be the rational explanation why even bright people who entered ST do not find a way out. Many decades of work which only produced metaphoric derivatives of ST as M-theory, extra dimensions, etc. have produced an avalanche of problems without solutions, so that what was claimed as "progress" only consisted of producing even more bizarre problems. In no epoch of particle theory was the relation between problem production and problem solution as extreme as presently.

The mathematical content of ST is the construction of a 10 parametric infinite component one particle representation of the Poincaré group (the superstring representation) on the oscillator algebra of a supersymmetric 10 component chiral current model. This is an unexpected and therefore interesting mathematical fact since those QFT models which are relevant for particle physics always describe in addition to discrete one-particle states a scattering continuum. It was pointed out in section 2 that this is the only known solution of a project formulated by Majorana: the search of an irreducible algebraic structure which contains a relativistic infinite component field equation. Several particle theorists, who were pursuing the same project in the 60s and ended (as Majorana) empty-hand, never noticed that ST found precisely one realization.

Even if the string theorists had noticed that by chance they discovered a solution to an old problem, they would have been hardly interested since they wanted to advance Mandelstam's dynamic S-matrix-based on-shell project. Instead they got stuck with the picture puzzle aspect of the relation between the (d, s) scale-dimension spectrum in a conformal QFT and (m^2, s) Poincaré group representation spectrum. This created the impression of having come across a deep new theory which describes not only the interaction of known particles but also contained the theory of gravity (a theory of everything). The problem is that with only mathematical rigor, but no conceptual frame which reveals the connection between the causality principles expressed in terms of fields or operator algebras, it is not really possible to formulate an S-matrix driven on-shell project. On the other hand, not knowing these subtle properties or not being bound by them, one runs the danger of creating a rich mathematical consistent physical fantasy world.

What made the situation even more muddled, is the fact that mathematicians were able to abstract from the rather loose pictures of the string theorists valuable mathematical ideas, which in many cases string theorists in turn took as a confirmation that they were working on a deep, albeit somewhat mysterious new physical theory.

The correct understanding of particle crossing was not possible without perceiving the new role of the S-matrix as *a relative modular invariant* between the free incoming and the interacting wedge-localized subalgebra. In this way the derivation of the particle crossing identity became an important part of in the formulation of a new constructive top-to-bottom approach which starts from the classification and construction of generators of wedge-localized algebras from a known S-matrix[41] and aims to construct the net of compact localized from nontrivial intersections. It was successful for integrable models, for which it leads to existence proofs and provides the setting for explicit calculations. Integrable models are limited to d=1+1 dimensions, but present an interesting theoretical laboratory for a future nonperturbative access to general models of QFT.

A comprehensive analysis of the causes of the existing deep schism within particle theory is not possible without looking also at sociological aspects. The appearance of wrong or useless theoretical constructs in a highly speculative area as particle theory is nothing new; the real problem is to understand why the dual model and ST, unlike numerous other failed ideas ("peratization", "Reggeization", SU(6), infinite component fields, Lee-Wick theory,...), succeeded to hold on for almost 5 decades despite demonstrable conceptual misunderstandings. A possible answer is that the memory about its conceptual roots as an *on-shell construction project in local quantum physics* got lost after almost 5 decades. The main damage caused by ST to particle theory is not coming from its errors but rather from the confusion it created about true string-localization whose undestanding is pivotal to formulate renormalizable interactions for any spin, in particular for massive vectormesons (section 3).

When, as a result of new ideas about analytic on-shell properties and their algebraic formulation from modular localization, on-shell construction ideas returned at the end of the 90s, string theory had already lost its connection to its own roots. Already during the 80s, ST begun meandering through conferences and journals by creating its own label in order to disconnect itself from its roots from in the strong interaction S-matrix project of the 60s. In this deliberate ahistorical self-presentation, it succeeded to convince many newcomers to particle theory that it presents the wave of the future (theory of everything), with QFT being assigned the role of a footnote. The conceptual differences between new foundational insights about on-shell constructions (as presented in the present paper) and ST became irreconcilable.

Among the theoreticians who followed the foundational developments of local quantum physics it is hard to meet anybody who is not aware that ST and most of its derivatives (the Maldacena conjecture, brane physics, embeddings of one QFT into a higher dimensional one and its inverse: dimensional Kaluza-Klein reduction) are results of a conceptual

[41] Apart from the bootstrap construction of scattering functions for integrable models, the construction of an S-matrix cannot be separated from the construction of the wedge generator using the sytem of equations which follow from wedge duality. The hope is that the combined on-shell equations, unlike the standard off-shell perturbation theory, permit a convergent iteration which determines the S-matrix together with the wedge generators.

flaw which resulted from a muddled view about localization in QFT in comparison to QM. Attempts to explain to string theorists why the Maldacena conjecture is incorrect in terms of the impossibility to encounter *physically* acceptable models on both sides of the AdS-correspondence end always in impasse; either because the concepts used are outside the understanding of ST, or the discussion ends by claiming that the "German correspondence" (referring to Rehren's theorem) has no bearing on Maldacena's conjecture. For the first time in the history of particle physics a whole community got into a standoff situation in which its conceptual resources are insuffient to liberate themselves from their self-created scientific isolation.

Perhaps the schism has even deeper philosophical roots in the way particle research was conducted. Since Dirac's successful extraction of antiparticles from the later abandoned "hole theory", the method of research consisted in starting a computation and thinking about necessary modification "as one moves along". Often correct discoveries were made in settings which later turned out to be incorrect. This trial and error method was for several decades extremely successful; most of the impressive results in particle physics after world war II were obtained in this way. More foundational directed research projects also existed parallel to this mainstream method; the oldest project was Wigner's 1939 classification of one-particle wave function spaces in terms of the representation theory of the Poincaré group, followed by Wightman's operator-valued distribution setting (shortly after Laurant Schwartz pathbreaking mathematical work on singular functions) and by Haag's 1957 formulation of "local quantum physics" in terms of nets of localized operator algebras. But there was little mainstream motivation for getting interested in such problems, as long as the "compute, think and correct" way of conducting research was successful[42].

Cul de sac situations occasionally caused by ideas with little or no foundational support were usually cleared up within the well-functioning traditional European "Streitkultur" (represented by great figures as Pauli, Jost, Lehmann, Kallen, Landau,..) which at that time also took roots in the US (Oppenheimer, Feynman, Schwinger, Dyson,..). But this way of keeping viable progress going disappeared in the 70s. It may not be accidental, that after developing the Standard Model within the setting of gauge theories, the rate of genuine progress slowed down despite an increase in publications. In fact most of the problems one confronts nowadays (the Higgs issue, long distance behavior, the precise meaning of asymptotic freedom,...) were formulated and discussed in the 70s. This suggests that the mentioned method of conducting research in particle theory may not be working anymore, and that time has come for a new conceptual push. Fortunately at this time one is not empty-handed, LQP has matured and is now ready to make contact with important unsolved problems of interactions involving higher spins; the ideas in section 3 illustrate this point.

Nowhere has this dispute about the future of particle theory taken such extreme ideological forms as that about Maldacena's conjecture concerning the physical content of the AdS-CFT correspondence. As explained in section 3.4, the correct mathematical statement is that there is indeed an algebraic isomorphism, but that its physical content is severely limited by the fact that (depending on what side one starts), either the resulting

[42]In more recent times Tegmark [97] proposed a more radical motto about how to conduct research in ST and its derivatives.

CFT violates the causal completion property (leading to the from nowhere into the causal shadow entering of "poltergeist" degrees of freedom, see section 2), or the degrees of freedom of the resulting AdS theory remain below the cardinality of phase space degrees of freedom which is necessary to obtain nontrivial compactly localized subalgebras ("anemia" of degrees of freedom to populate a larger spacetime region). This is of course in agreement with the impossibility to illustrate the phenomenon in terms of explicit models, since Lagrangian quantization is formally in agreement with both aspects of causality. Although Lagrangian quantization cannot reveal the mathematical existence since the renormalized series diverges, one believes that its structural properties correctly mirror foundational properties of QFT.

The insistence in the correctness of the Maldacena conjecture and the public use of derogatory terminology as "the German correspondence" for the proven theorem marks the sociological depth of the schism. For most particle physicists with an awareness about the past of their subject it is of course somewhat sad to see that the insights gained in pre-electronic times into the connection between the causal completion property and the cardinality of phase space degrees of freedom (see section 2) have succumbed to the maelstrom of time in regard to the string-inspired generation. These insights had been obtained at a time when progress was still available without such foundational knowledge was not necessary since progress was still forthcoming without knowledge about QFT. But now, when these post-quantization results are really needed (see section 3) they are not available to the protagonists of the above conjecture and related subjects.

The situation is not so dissimilar from that in the financial markets; at the time when the tools of deregulated capitalism were working, hardly anybody was interested to listen to alternatives for what to do when one day they start tearing society apart. Apparently not even surreal consequences [81] are able to prevent people from being addicted to wrong conjectures as long as there is a sufficiently large community of subscribers. the only occurrence which would be the beginning of the end of the ST and its derivatives is if one of its main supporters and updaters begins to have scrupels about what they are doing to science; but after more than 30 years of investments in ST this is even less probable than a banker developing doubts about the ethical aspects of financial capitalism.

There is hardly anything more bizarre than the idea that we are living in a dimensionally reduced 10 dimensional target space of a chiral conformal QFT. Attributing to this observation the role of a key for understanding of the universe is not much different than the ontological role *attributed to the number 42 as an answer to the ultimate question about "Life, the Universe, and Everything"* in Douglas Adam's well-known scientific fiction comedy "the hitchhiker's guide through the galaxy".

In a way it is very fitting that a prize, which has been donated by somebody [82] who profited from this kind of capitalism, is given to the kind of unproductive but entertaining ST influenced particle theory which is sustained by ignorance about prior foundational results. It opens the possibility to physicists to get rich in the same way as the financial players to which the sponsor of this prize belongs, namely by creating unproductive toxic, and in case of particle physics, bizarre inventions. Usually the critique against giving highly lucrative prizes to less than Nobel worthy observations can be dismissed as resulting from envy of the person from whom the critique originated. But if the scientific results are at variance with known facts such prizes reveal to critical minds and future historians serious doubts about the state of health about whether particle physics is able to maintain its status it had since the days of Einstein, Heisenberg and others also also in the new social surrounding of rampant capitalism.

Never in the history of physics before has an area of research lend itself that easily to be used in entertainment and cinema as ST and its bizarre but entertaining derivatives as extra dimensions and dimensional reduction [85]; the ST saga has been spread worldwide on television by the former string-theorist Brian Green [86].

To feel the depth of the crisis into which large parts of particle theory has fallen, it is helpful to be reminded of a quotation from Einstein's talk in the honor of Planck [87].

In the temple of science are many mansions, and various indeed are they who dwell therein and the motives that have led them thither. Many take to science out of a joyful sense of superior intellectual power; science is their own special sport to which they look for vivid experience and the satisfaction of ambition; many others are found in the temple who have offered the product of their brains on this altar for purely utilitarian purposes. Were an angel of the Lord to come and drive all these people belonging to these two categories out of the temple, the assemblage would be seriously depleted, but there would still be some men, of present and past times, left inside. Our Planck is one of them, and that is why we love him. ...

But where has Einstein's *Angel of the Lord*, the protector of the temple of science, gone in the times of string theory and all its derivatives? With the continuation of the old Streitkultur we would have had a chance to get out of this, in fact we may not even have gotten into it.

Given that sociological situation with respect to ST and its derivatives, one should not expect to get out of ST in the foreseeable future. It is more probably that the ongoing progress about renormalization theory from string-localized higher spin fields, in particular new insights about renormalization of vectorpotentials in massive and massless s=1 models (indicated in section 3), could achieve such a revolution.

In the past it was easy to ignore the existing critical remarks, since no concerted efford at a *scientific* critique of ST existed; people just expressed opinions about its bizarre consequences or pointed to the decades passed without any experimentally accessible consequences (Woit) and to other sociological-philosophical points (Smolin). Actually in an older paper Smolin together with Arnsdorf came quite close to raise an important scientific point [88]. These two authors, standing on the shoulders of Rehren, pointed at a kind of conundrum between the consequences of the string-induced Maldacena conjecture [81] and Rehren's theorem [52]. This is precisely connected to the degrees of freedom problem explained in section 2 and 3.

One can ask the question whether it is possible to modify the AdS-CFT setting, so that an appropriately reformulated Maldacena's conjecture can be saved from the enormous pile of publications by establishing harmony with the rigorous theorem. This is precisely the question Kay and Ortiz asked [89]. Taking their cue from prior work on the correspondence principle of Mukohyama-Israel as well from 't Hoofts brick-wall idea[43] [90], these authors start with a Hartle-Hawking-Israel like pure state on an imagined

[43]This idea seems to imply a conjecture about the dependence of localization entropy of a fuzzy surface which is expected to result as a theorem from the degrees of freedom picture which leads to the split property.

combined matter + gravity dynamic system. They then propose to equate the AdS side of a hypothetical conformal invariant supersymmetric Yang-Mills model with the restriction of the H-H-I state to a matter subsystem which is in accordance with Rehren's theorem. That this can be achieved is not very plausible (as the authors themselves admit).

Concerning defences of ST, one may refer to a recent paper by Duff [91] "String and M-Theory: Answering the Critics" within a project "Forty Years Of String Theory" where the author basically musters all the names of well-known people (besides the hard core string theorists) who, guided by their natural intellectual curiosity looked at ST and whose first (and in most cases only) reaction was quite positive. Feynman's name does not appear there, which may be related to his well-known accusation of string theorist to counter scientific critique by inventing excuses.

Further critical remarks will be left to the philosophers and historians of physics; the 50 years of unopposed derailment of parts of particle theory will provide ample material to be analyzed. The future potentiality of QFT stands in contrast to the present sociological-caused paralyzing schism within particle theory. Hopefully the present work succeeds to draw attention to the enormous potential, which the good old QFT still has in store for us if we are willing to engage in a pursuit of its foundations.

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References

[1] A. Duncan and M. Janssen, *Pascual Jordan's resolution of the conundrum of the wave-particle duality of light*, arXiv:0709.3812

[2] B. Schroer, *The Einstein-Jordan conundrum and its relation to ongoing foundational research in local quantum physics*, to be published in EPJH, arXiv:1101.0569

[3] R. Haag, *Local Quantum Physics*, Springer 1996

[4] W. G. Unruh, *Notes on black hole evaporation*, Phys. Rev. **D14**, (1976) 870

[5] G. Sewell, Ann. Phys. **141**, (1982) 201

[6] D. Buchholz and C. Solveen, *Unruh Effect and the Concept of Temperature*, arXiv:1212.2409

[7] B. Schroer and H.-W. Wiesbrock, Rev. Math. Phys.**12** (2000) 461, arXiv:hep-th/9901031

[8] R. Kaehler and H.-P. Wiesbrock, *Modular theory and the reconstruction of four-dimensional quantum field theories*, Journal of Mathematical Physics **42**, (2001) 74

[9] B. Schroer, *Bondi-Metzner-Sachs symmetry, holography on null-surfaces and area proportionality of "light-slice" entropy*, Foundations of Physics **41**, 2 (2011), 204, arXiv:0905.4435

[10] B. Schroer, *The foundational origin of integrability in quantum field theory*, to be published in Foundations of Physics, arXiv:1109.1212

[11] B. Schroer, *Studies in History and Philosophy of Modern Physics* **41** (2010) 104–127, arXiv:0912.2874

[12] H. Epstein and V. Glaser, Ann. Inst. Henri Poincaré A XIX, (1973) 211

[13] W. Heisenberg, *Die beobachtbaren Grössen in der Theorie der Elementarteilchenphysik*, Z. Physik **120**, (1943) 513

[14] J. Bros, H. Epstein and V. Glaser, Com. Math. Phys. **1**, (1965) 240

[15] S. Mandelstam, Phys. Rev. **112**, (1958) 1344 and Phys. Rev. **115**, (1959) 1741

[16] G. Veneziano, Nuovo Cim. A **57**, (1968) 190

[17] G. Mack, *D-dimensional Conformal Field Theories with anomalous dimensions as Dual Resonance Models*, arXiv:0907.2407

[18] G. Mack, *D-independent representation of Conformal Field Theories in D dimensions via transformation to auxiliary Dual Resonance Models. Scalar amplitudes*, arXiv:0907.2407v1

[19] R. Brunetti, D. Guido and R. Longo, *Modular localization and Wigner particles*, Rev. Math. Phys. **14**, (2002) 759

[20] J. Mund, B. Schroer and J. Yngvason, *String-localized quantum fields and modular localization*, CMP **268** (2006) 621, math-ph/0511042

[21] H-J. Borchers, *On revolutionizing quantum field theory with Tomita's modular theory*, J. Math. Phys. **41**, (2000) 8604

[22] B. Schroer, Annals Phys. **275** (1999) 190

[23] G. Lechner, *An Existence Proof for Interacting Quantum Field Theories with a Factorizing S-Matrix*, Commun. Mat. Phys. **227**, (2008) 821, arXiv.org/abs/math-ph/0601022

[24] R. Haag, *Discussion of the 'axioms' and the asymptotic properties of a local field theory with composite particles* (historical document), Eur. Phys. J. H 35, 243–253 (2010)

[25] K. Fredenhagen, *Lille 1957: The birth of the concept of local algebras of observables*, Eur. Phys. J. H 35, 239–241 (2010)

[26] B. Schroer, *Pascual Jordan's legacy and the ongoing research in quantum field theory*, Eur. Phys. J. H **35**, (2011) 377, arXiv:1010.4431

[27] R. F. Streater and A. S. Wightman, PCT Spin&Statistics and all that, New York, Benjamin 1964

[28] R. Haag and B. Schroer, *Postulates of Quantum Field Theory*, J. Mat. Phys. **3**, (1962) 248

[29] D. Buchholz and E. Wichmann, Commun. Math. Phys. **106** (1986) 321

[30] D. Buchholz, C. D'Antoni and R. Longo, *Nuclear maps and modular structures. I. General properties*, J. Funct. Anal. **88** (1990) 233

[31] G. C. Hegerfeldt, Causality problems in Fermi's two atom system, Phys. Rev. Lett. **72**, (1994) 596-599

[32] D. Buchholz and J. Yngvason, *There are no causality problems for Fermi's two atom system*, Physical Review Letters **73**, (1994) 613-616

[33] B. Schroer, *An alternative to the gauge theory setting*, Found. of Phys. **41**, (2011) 1543, arXiv:1012.0013

[34] B. Schroer, *Unexplored regions in QFT and the conceptual foundations of the Standard Model*, arXiv:1010.4431

[35] S. Hollands and R. M. Wald, General Relativity and Gravitation **36**, (2004) 2595

[36] J Polchinski, *String theory I*, Cambridge University Press 1998

[37] D. Buchholz, *New Light on Infrared Problems: Sectors, Statistics, Spectrum and All That*, arXiv:1301.2516

[38] E. Majorana, Teoria relativistica di particelle con momentum internisico arbitrario, Nuovo Cimento 9, (1932) 335

[39] N. N. Bogoliubov, A. Logunov, A. I. Oksak and I. T. Todorov, *General principles of quantum field theory*, Dordrecht Kluwer

[40] D. A. Lowe, Phys. Lett. B 326, (1994) 223

[41] E. Martinec, Class. Quant. Grav. **10**, (1993) 187

[42] R. Jost: TCP-Invarianz der Streumatrix und interpolierende Felder, Helvetica Phys. Acta **36**, (1963) 77

[43] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. B **241** (2), (1984) 333

[44] M. Lüscher and G. Mack, Comm. Math. Phys. **41**, (1975) 203

[45] B. Schroer and J. A. Swieca, Phys. Rev. D **10**, (1974) 480

[46] P. Di Vecchia, *The birth of string theory*, Lect. Notes Phys.**737**, (2008) 59-118, arXiv 0704.0101

[47] B. Schroer, *Causality and dispersion relations and the role of the S-matrix in the ongoing research*, Foundations of Physics **42**, (2012) 1481, arXiv:1107.1374

[48] D. Buchholz, G. Mack and I. Todorov, Nucl.Phys. B, Proc. Suppl. **5B**, (1988) 20

[49] C.P. Staskiewicz, Die lokale Struktur abelscher Stromalgebren auf dem Kreis, Thesis at Freie Universitaet Berlin (1995)

[50] Y. Kawahigashi and R. Longo, Adv. Math. **206** (2006) 729, arXiv:math/0407263

[51] D. Buchholz and K. Fredenhagen, Commun. Mat. Phys. **84**, (1982) 1

[52] K-H. Rehren, *Algebraic Holography*, Annales Henri Poincaré**1**, (2000) 607, arXiv:hep-th/9905179

[53] D. Bahns, K. Rejzner and J. Zahn, *The effective theory of strings*, arXiv:1204.6263

[54] K. Fredenhagen and K. Rejzner. *Local covariance and background independence*, arXiv:1102.2376

[55] D. Bahns, J. Math. Phys. **45**, (2004) 4640

[56] S. Weinberg, *The Quantum Theory of Fields I,* Cambridge University Press

[57] J. Yngvason, Zero-mass infinite spin representations of the Poincar´e group and quantum field theory, Commun. Math. Phys. **18** (1970), 195

[58] M. Plaschke and J. Yngvason, Journal of Math. Phys. **53**, (2012) 042301

[59] S. Mandelstam, Phys. Rev. **175**, (1968) 1518

[60] K. Bardackci and B. Schroer, *Local Approximations in Renormalizable and Nonrenormalizable Theories II*, J. Math. Phys **7**, (1966) 16.

[61] G. Björck, Arkiv för Mathematik **6**, (1965) 351

[62] A. Jaffe, Phys. Rev. **158**, (1967) 1454

[63] J. Mund, *String-localized massive vector Bosons without ghosts: the example of massive spinor QED*, to appear

[64] B. Schroer, *Interactions with quadratic dependence on string-localized massive vectormesons: the example of scalar QED*, in preparation

[65] G. Scharf, Quantum Gauge Theories, a true ghost story, John Wiley&Sons, INC 2001

[66] J. H. Lowenstein and B. Schroer, Phys. Rev. **D7**, (1975) 1929

[67] L. Fassarella and B. Schroer, *Wigner particle theory and local quantum physics*, J. Phys. A **35**, (2002) 9123-9164

[68] B. Schroer, *Modular localization and the bootstrap-formfactor program*, Nucl. Phys. **B499**, 1997, 547, hep-th/9702145.

[69] B. Schroer, *Modular localization and the d=1+1 formfactor program*, Annals of Physics **295**, (1999) 190

[70] R. Kaehler and H.-P. Wiesbrock, *Modular theory and the reconstruction of four-dimensional quantum field theories*, Journal of Mathematical Physics **42**, (2001) 74-86

[71] B. Schroer, *Particle physics in the 60s and 70s and the legacy of contributions by J. A. Swieca*, Eur.Phys.J.H **35**, (2010) 53, arXiv:0712.0371

[72] J. Schwinger, Trieste Lectures, 1962, p. 89, I.A.E.A., Vienna 1963

[73] A. J. Swieca, *Charge screening and mass spectrum*, Phys. Rev. D **13**, (1976) 312

[74] M. Duetsch, J. M. Gracia-Bondia, F. Scheck, J. C. Varilly, *Quantum gauge models without classical Higgs mechanism*, arXiv:1001.0932

[75] J. Mund, *An Algebraic Jost-Schroer Theorem for Massive Theories*, arXiv:1012.1454

[76] H. J. Borchers, D. Buchholz and B. Schroer, Commun. Math. Phys. **219** (2001) 125

[77] H. Babujian, A.Fring, M. Karowski and A. Zapletal, Nucl. Phys. B **538**, (1990) 535

[78] Buchholz and S.J Summers, *Scattering in Relativistic Quantum Field Theory: Fundamental Concepts and Tools*, arXiv:math-ph/0405058

[79] D. Buchholz, Commun. math. Phys. **36**, (1974) 243

[80] G. Lechner, *Deformations of quantum field theories and integrable models*, arXiv:1104.1948

[81] J. Maldacena, *The Illusion of Gravity*, Scientific American, (November 2005) page 56-63

[82] http://en.wikipedia.org/wiki/Fundamental_Physics_Prize

[83] J. Bros and J. Mund, *Braid group statistics implies scattering in three-dimensional local quantum physics*, arXiv:1112.5785

[84] http://golem.ph.utexas.edu/~distler/blog/archives/000987.html

[85] L. Randall on http://www.youtube.com/watch?v=Q5ikGoJTpMM

[86] http://www.ted.com/talks/brian_greene_on_string_theory.html

[87] http://www.scribd.com/doc/13093631/Einstein-in-His-Own-Words

[88] M. Arnsdorf and L. Smolin, *The Maldacena Conjecture and Rehren Duality*, arXiv:hep-th/0106073

[89] B. S. Kay and L. Ortiz, *Brick Walls and AdS/CFT*, arXiv:1111.6429

[90] G. 't Hooft, *On the quantum structure of a black hole*, Nucl. Phys. B **256**, (1985) 727

[91] M. J. Duff, *String and M-theory: answering the critics*, arXiv:1112.0788

[92] H. Babujian and M. Karowski, Int. J. Mod. Phys. **A1952**, (2004) 34, and references therein to the beginnings of the bootstrap-formfactor program

[93] J. Mund, Prog. Math. **251**, (2007) 199, arXiv:hep-th/0502014

[94] J. Mund, private communication

[95] R. M. Wald, *The Formulation of Quantum Field Theory in Curved Spacetime*, arXiv:0907.0416

[96] R. Haag and J. A. Swieca, *When does a quantum field theory describe particles?*, Math. Phys. **1**, (1965) 308

[97] M. Tegmark, *Shut up and calculate*, arXiv:0709.4024, and prior similar philosophical Zeitgeist contributions