Relations between Positivity, Localization and Degrees of Freedom:

the Weinberg-Witten theorem and the van Dam-Veltman-Zakharov discontinuity

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Abstract

The problem of accounting for the degrees of freedom in passing from massive spin to massless helicity degrees of freedom and its inverse, the "fattening" of massless tensor fields to their massive $s = |h|$ counterparts, are solved using "string-localized fields".

This approach allows to overcome the Weinberg-Witten impediment against the existence of massless $|h| \geq 2$ energy-momentum tensors, and to replace the van Dam-Veltman-Zakharov discontinuity concerning, e.g., very light gravitons, by a continuous limit $m \to 0$.

1 Introduction

In relativistic quantum field theory, the quantization of interacting massless or massive classical potentials of higher spin ($s \geq 1$) either violates Hilbert space positivity which is an indispensable attribute of the probability interpretation of quantum theory, or leads to a violation of the power counting bound of renormalizability whose maintenance requires again a violation of positivity.

In order to save the positivity for those quantum fields which correspond to classically gauge invariant observables one has to formally extend the theory by adding degrees of freedom in the form of negative metric Stückelberg fields and "ghosts" without a counterpart in classical gauge theories. The justification for this quantum gauge setting is that one can extract from the indefinite metric Krein space a Hilbert space which the gauge invariant operators generate from the vacuum.

This situation is satisfactory as far as the perturbative construction of a unitary gauge-invariant S-matrix is concerned. However the theory remains incomplete in that it provides no physical interpolating fields which mediate between the causal localization principles of the field theory and the analytic structure of the S-matrix. Expressed differently, gauge theory allows to compute the perturbative S-matrix, but cannot construct its off-shell extension on a Hilbert space.

There are two famous results about the higher-spin massless case. The first is the Weinberg-Witten theorem [21] which states that for $s \geq 2$, no point-localized stress-energy tensor exists such that the Poincaré generators are moments of its zero-components. This result also obstructs the semiclassical coupling of massless higher spin matter to gravity.

The second is the DVZ observation due to van Dam and Veltman [22] and to Zakharov [24], that in interacting models with $s \geq 2$, scattering amplitudes are discontinuous in the mass at $m = 0$, i.e., the scattering on massless gravitons (say) is significantly different from the scattering on gravitons of a very small mass.

Both problems, and the positivity problem of gauge theories, can be solved with the help of "string-localized quantum fields" defined in the physical Hilbert space. These are potentials defined as integrals over their field strengths (and derivatives thereof) with the same particle content. String-localized massive potentials of spin s having an improved UV dimension $d_{\mathrm{UV}} = 1$ rather than $d_{\mathrm{UV}} = s + 1$, admit renormalizable interactions that are otherwise excluded by power-counting.

A point-localized massive spin s potential can be split up into a string-localized potential that has a massless limit, and derivatives of one or more so-called "escort fields". The role of the latter is to separate off derivative terms from the interaction Lagrangean or from conserved currents, that do not contribute to the S-matrix or to charges and Poincaré generators, respectively. They thus "carry away" all non-renormalizable UV fluctuations and singularities as $m \to 0$.

How this works, may be illustrated in the case of QED [18, 19, 10]: The coupling to the indefinite Maxwell potential A^{K} ("K" stands for "Krein") is replaced by a coupling $j^\mu A^{\mathrm{P}}_\mu$ to the massive Proca potential A^{P}. This avoids negative-norm states, but the interaction is non-renormalizable because of the UV dimension 2 of the Proca potential. Now, the decomposition (see Sect. 2) $A^{\mathrm{P}}_\mu(x) = A_\mu(x, e) - m^{-1}\, \partial_\mu a(x, e)$ into a string-localized potential and its escort comes to bear: $A_\mu(e)$ has UV dimension 1 and is regular at $m = 0$. The UV-divergent part of the interaction is carried away by the escort field: $-m^{-1}\, j^\mu \partial_\mu a(e) = -\partial_\mu(m^{-1}\, j^\mu a(e))$ is a total derivative and may be discarded from the interaction Lagrangean. The remaining string-localized (but equivalent to the point-localized) interaction $j^\mu A_\mu(e)$ has UV dimension 4, and can be taken at $m = 0$.

The ongoing analysis of perturbation theory with string-localized interactions [7, 10, 14] gives strong evidence that the resulting theory is order-by-order renormalizable, and equivalent to the "usual" QED. The scattering matrix can be made independent of the string direction e, provided a suitable renormalization condition is satisfied. This condition can be seen as an analogue of Ward identities stipulating BRST invariance in point-localized but indefinite approaches [17, 4]. Indeed, the condition can also be formulated in a cohomological manner. Yet, the precise relation between gauge invariance and string-independence remains to be explored.

Whereas string-localized perturbation theory is still in its infancy, the problems of massless currents and energy-momentum tensors as well as the continuous passage from free massive fields to their massless helicity counterparts can be completely solved. The presentation of this solution is the principal aim of this letter, including also the opposite direction, sometimes (in connection with the Higgs mechanism) referred to as "fattening".

1.1 Overview of results

We outline the general picture for arbitrary integer spin s, referring to [12] for further details. As the case $s = 2$ exhibits all the features of the general case, we focus ourselves to $s = 1$ and $s = 2$ in Sect. 2, Sect. 3.

The 2-point functions of covariant massless potentials are indefinite polynomials in the metric tensor $\eta_{\mu\nu}$, while their field strengths (curl in all indices) are positive. (By "positive", it is understood "positive-semidefinite", accounting for null states due to equations of motion like $\partial^\mu F_{\mu\nu} = 0$.) The field strengths can also be constructed, without reference to a potential, directly on the Fock space over the unitary massless helicity $h = \pm 1$ Wigner representation of the Poincaré group. This is exposed in standard textbooks, e.g., [20]. One can construct potentials in the Coulomb gauge on the same Hilbert space, but one gets into conflict with Poincaré covariance: Lorentz transformations result in an operator-valued gauge transformation due to the affine nature of the Wigner phase. When the potentials are required for interactions, and one has to compromise between positivity or Lorentz invariance, preference is usually given to covariance.

For some early treatments of massive free tensor fields of higher spin, see [3, 6]. We freely adopt the name "Proca" for all spins $s \geq 1$. The Proca potentials are symmetric traceless and conserved tensors $A^{\rm P}_{\mu_1\dots\mu_s}(x)$ of rank s. Their 2-point functions obtained from the (m, s) Wigner representation [20] are polynomials in the positive projection orthogonal to the momentum (sign convention $\eta_{00} = +1$)

$$-\pi_{\mu\nu}(p) = -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

with coefficients dictated by symmetry and tracelessness. The momenta in the numerator cause the UV dimension $d_{\rm UV} = s + 1$ and, by power counting, jeopardize the renormalizability of minimal couplings to currents.

The potentials admit no massless limit. Only their field strengths $F_{[\mu_1\nu_1]\dots[\mu_s\nu_s]}$ exist at $m = 0$ because the curls kill the terms with momentum factors.

We define symmetric tensor fields $A^{(r)}_{\mu_1\dots\mu_r}$ of rank $0 \leq r \leq s$ on the Fock space of the massive field strengths such that

- All $A^{(r)}$ have UV dimension $d_{\rm UV} = 1$ and are regular in the massless limit.

• The potential $A^{\rm P}$ can be decomposed in a way that (i) all contributions of UV dimension > 1 are isolated as derivatives of the escort fields $A^{(r)}$ of lower rank $r < s$, and (ii) the singular behaviour at $m \to 0$ is manifest in the expansion coefficients (inverse powers of m).

• The massive fields $A^{(r)}$ are coupled among each other through their traces and divergences. In the massles limit, they become traceless and conserved, and their field equations and 2-point functions decouple.

• At $m = 0$, the escort $A^{(0)}$ is the canonical massless scalar φ. The tensors $A^{(r>0)}$ are potentials for the field strengths of helicity $h = \pm r$ [20]. They were previously constructed [16] without an approximation from $m > 0$.

• Conversely, the given massless potential $A^{(s)}$ of any helicity $h = \pm s$ can be made massive ("fattening") by simply changing the dispersion relation $p^0 = \omega_m(\vec{p})$. The fattened field brings along with it all lower rank fields $A^{(r)}$ by virtue of the coupling through the divergence. We give a surprisingly simple formula involving only derivatives, to restore the exact Proca potential $A^{\rm P}$.

• We construct a stress-energy tensor for the massless fields that decouples into a direct sum of mutually commuting stress-energy tensors $T^{(r)}$ for the helicity potentials $A^{(r)}$.

The massless limit describes the exact splitting of the (m, s) Wigner representation into massless helicity representations with $h = \pm r$ $(r = 1, \dots, s)$ and $h = 0$.

In particular, the number $2s + 1$ of one-particle states at fixed momentum is preserved. In contrast, the "fattening" of the massless helicity s field increases the number of one-particle states, because its 2-point function is a semi-definite quadratic form of rank 2 that becomes rank $2s + 1$ under the deformation of the dispersion relation.

These facts yield an obvious explanation of the DVZ discontinuity [22, 24]: The spin 2 Proca potential $A^{\rm P}$ (or its analog in the indefinite Feynman gauge) is not continuously connected with a massless helicity $h = \pm 2$ potential. At each positive mass, the former has contributions from all $r \leq 2$. Rejecting at $m = 0$ the helicities $|h| < 2$ causes the discontinuity. A coupling through $A^{(2)}$ at every mass would instead smoothly decrease the contributions of the lower helicities.

The stated properties of the massless potentials and stress-energy tensors are clearly at variance with many No-Go theorems, including the Weinberg-Witten theorem. This is possible because they are string-localized. Their 2-point functions involve, instead of the singular (as $m \to 0$) tensor $\pi_{\mu\nu}(p)$ or indefinite tensor $\eta_{\mu\nu}$, a suitable tensor $E_{\mu\nu}(p)$ whose substitution into the 2-point functions (i) preserves positivity, (ii) does not affect the field strengths, and (iii) has a regular limit $m \to 0$.

The No-Go theorems may be accounted to the fact that such a tensor $E_{\mu\nu}(p)$ does not exist, if it is allowed to be a function of the momentum only. Instead,

$$E(e, e')_{\mu\nu}(p) := \eta_{\mu\nu} - \frac{p_\mu e_\nu}{(pe)_+} - \frac{e'_\mu p_\nu}{(pe')_+} + \frac{(ee')p_\mu p_\nu}{(pe)_+(pe')_+}$$

(where $i/(k)_+ = i/(k + i0)$ is the Fourier transform of the Heaviside function) are distributions in p and two four-vectors e, e'. If $E_{\mu\nu}$ is substituted for $\pi_{\mu\nu}$ or $\eta_{\mu\nu}$, the potentials depend on e, but the field strengths will not.

In momentum space, the integration

$$X(x,e) \equiv (I_eX)(x) := \int_{\mathbb{R}_+} d\lambda\, X(x+\lambda e) \tag{1.1}$$

produces the denominators $i((pe)+i0)^{-1}$ in the creation part and $-i((pe)-i0)^{-1}$ in the annihilation part. Thus, fields whose 2-point functions are polynomials in $E_{\mu\nu}$ are necessarily string-localized.

String-localization requires some comments. First, it is not a feature of the associated particles, but of the fields used to couple them to other particles. (The only exception are particles in the infinite-spin representations [15, 8], that are beyond the scope of this letter.)

Eq. (1.1) (and its generalizations involving several integral operations I_e) imply the Poincaré transformations of string-localized fields

$$U_{a,\Lambda} A_{\mu_1\dots\mu_r}(x,e) U^*_{a,\Lambda} = \big(\prod\nolimits_i \Lambda^{\nu_i}{}_{\mu_i}\big) A_{\nu_1\dots\nu_r}(a+\Lambda x, \Lambda e),$$

i.e., the direction of the string is transformed along with its apex x and the tensor components of the field tensor.

There is no conflict with the principle of causality, which is as imperative in relativistic quantum field theory as Hilbert space positivity. String-localized fields satisfy causal commutation relations according to their localization: two fields commute whenever their strings are pointwise spacelike separated. There are sufficiently many spacelike separated pairs of spacelike or lightlike strings to construct scattering states by asymptotic cluster properties (Haag-Ruelle theory).

String-localized interactions admit couplings of physically massive tensor potentials without spontaneous symmetry breaking. Instead, when coupling self-interacting massive vector bosons (like W and Z bosons) via their string-localized potentials, the string-independence can only be achieved with the help of a boson with properties like the Higgs, including a quartic self-interaction [19]. Its role is not the generation of the mass, but the preservation of the renormalizability and locality.

Examples of new renormalizable interactions in the string-localized setting could be the coupling of matter to (massive) gravitons through the string-localized potentials $A^{(2)}$, and perhaps the self-coupling of gravitons.

In the sequel, we give more details for spin 1 and 2. All displayed linear relations between fields follow from their definitions by integrals and derivatives of point-localized fields, e.g., by inspection of their integral representations in terms of creation and annihilation operators.

We write 2-point functions throughout as

$$(\Omega, X(x)Y(y)\Omega) = \int d\mu_m(p)\cdot e^{-ip(x-y)}\cdot {}_mM^{X,Y}(p),$$

where $d\mu_m(p) = \frac{d^4p}{(2\pi)^3}\delta(p^2-m^2)\theta(p^0)$.

2 Spin one

The 2-point function of the massless Krein potential

$$ {}_0M^{A^{\rm K}_\mu,A^{\rm K}_\nu} = -\eta_{\mu\nu} $$

is indefinite. Its curl $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the Maxwell field with positive 2-point function

$$ {}_0M^{F_{\mu\nu},F_{\kappa\lambda}} = -p_\mu p_\kappa\,\eta_{\nu\lambda}+p_\mu p_\lambda\,\eta_{\nu\kappa}+p_\nu p_\kappa\,\eta_{\mu\lambda}-p_\nu p_\lambda\,\eta_{\mu\kappa}. $$

The massive Proca potential satisfies $\partial^\mu A^{\rm P}_\mu=0$. Its positive 2-point function is

$$ {}_mM^{A^{\rm P}_\mu,A^{\rm P}_\nu} = -\pi_{\mu\nu}(p). \tag{2.1} $$

The curl kills the term $p_\mu p_\nu/m^2$, so the field strength is regular at $m=0$. The field equation $\partial^\mu F_{\mu\nu}=-m^2A^{\rm P}_\nu$ gives back the potential in terms of its field strength.

Only for $s=1$, the massless limit can be achieved with point-localized fields, by inspection of their 2-point functions: $mA^{\rm P}_\mu$ is regular at $m=0$, where it decouples from $F_{\mu\nu}$ and becomes the derivative of the scalar free field φ with ${}_0M^{\partial_\mu\varphi,\partial_\nu\varphi}=p_\mu p_\nu$.

In the string-localized setting, the massless scalar emerges without derivative. We define

$$ \begin{aligned} A_\mu(x,e) &:= I_e\big(F_{\mu\nu}\big)(x)e^\nu \equiv \int_{\mathbb{R}_+} d\lambda\, F_{\mu\nu}(x+\lambda e)e^\nu, \\ a(x,e) &:= -m^{-1}\,\partial^\mu A_\mu(x,e). \end{aligned} \tag{2.2} $$

$A_\mu(e)$ is regular in the massless limit because $F_{\mu\nu}$ is. That $a(e)$ is also regular can be seen from

$$ {}_mM^{A_\mu(-e),A_\nu(e')} = -E(e,e')_{\mu\nu}(p), \tag{2.3} $$

which implies by the definition of $a(e)$

$$ {}_mM^{a(-e),A_\nu(e')} = O(m), \qquad {}_mM^{a(-e),a(e')} = 1+O(m^2) $$

(As the fields are distributions also in e [11], we have to admit independent string directions e, e'. The choice "$-e$" is a convenience paying off for higher spin [12].)

At $m=0$, the fields a and A_μ decouple, and converge to the massless scalar and (as the terms $O(p/(pe))$ in Eq. (2.3) do not contribute to $F_{\mu\nu}$) to a string-localized massless potential for the Maxwell field strength.

In addition, one gets the decomposition underlying the QED example in Sect. 1

$$ A^{\rm P}_\mu(x) = A_\mu(x,e)-m^{-1}\,\partial_\mu a(x,e). \tag{2.4} $$

The taming of the UV behaviour is seen from Eq. (2.3): the momentum factors in the denominators of $E(e,e')$ balance those in the numerators [11].

One can average the potential A_μ in e over the spacelike sphere with $e^0=0$. The resulting field is, at $m=0$, the Maxwell potential in the highly nonlocal [20] Coulomb gauge.

3 Spin two

The case $s = 2$ is largely analogous, but the decoupling at $m = 0$ requires a second step.

The positive 2-point function of the massless field strength $F_{[\mu\kappa][\nu\lambda]}$ can be represented as the curl of the indefinite 2-point function of the Krein potential

$$ {}_0M^{A^{\rm K}_{\mu\nu},A^{\rm K}_{\kappa\lambda}} = \frac{1}{2}\big[\eta_{\mu\kappa}\eta_{\nu\lambda} + \eta_{\mu\nu}\eta_{\kappa\lambda}\big] - \frac{1}{2}\eta_{\mu\nu}\eta_{\kappa\lambda}. \tag{3.1} $$

The coefficient $-\frac{1}{2}$ of the last term ensures that there are precisely two helicity states. The symmetric, traceless and conserved massive Proca 2-point function is

$$ {}_mM^{A^{\rm P}_{\mu\nu},A^{\rm P}_{\kappa\lambda}} = \frac{1}{2}\big[\pi_{\mu\kappa}\pi_{\nu\lambda} + \pi_{\mu\lambda}\pi_{\kappa\nu}\big] - \frac{1}{3}\pi_{\mu\nu}\pi_{\kappa\lambda}. \tag{3.2} $$

The coefficient $-\frac{1}{3}$ of the last term ensures the vanishing of the trace. The formulae for the massive and massless field strengths differ *only* by this coefficient. In particular, the massless field strength is not the limit of the massive field strength as $m \to 0$.

In the string-localized setting, we define the potential

$$ A_{\mu\nu}(x,e) := \big(I_e^2 F_{[\mu\kappa][\nu\lambda]}\big)(x)e^\kappa e^\lambda \tag{3.3} $$

and its escort fields

$$ \begin{aligned} a^{(1)}_\mu(x,e) &:= -m^{-1}\,\partial^\nu A_{\mu\nu}(x,e), \\ a^{(0)}(x,e) &:= -m^{-1}\,\partial^\mu a^{(1)}_\mu(x,e). \end{aligned} \tag{3.4} $$

Eq. (3.2) implies

$$ {}_mM^{A_{\mu\nu}(-e),A_{\kappa\lambda}(e')} = \frac{1}{2}\big[E(e,e')_{\mu\kappa}E(e,e')_{\nu\lambda} + (\kappa \leftrightarrow \lambda)\big] - \frac{1}{3}E(e,e)_{\mu\nu}E(e',e')_{\kappa\lambda}, \tag{3.5} $$

and one obtains the escort correlations with Eq. (3.4). The correlations between even and odd rank fields are $O(m)$ and decouple in the massless limit. The odd-odd and even-even correlations become

$$ \begin{aligned} {}_0M^{a^{(1)}_\mu(-e),a^{(1)}_\nu(e')} &= -\frac{1}{2}E(e,e')_{\mu\nu}(p), \\ {}_0M^{A_{\mu\nu}(-e),a^{(0)}(e')} &= -\frac{1}{3}E(e,e)_{\mu\nu}(p), \\ {}_0M^{a^{(0)}(-e),a^{(0)}(e')} &= \frac{2}{3}. \end{aligned} \tag{3.6} $$

$A_{\mu\nu}(e)$ and $a(e)$ do not decouple at $m = 0$, in fact one has $\eta^{\mu\nu}A_{\mu\nu}(e) = -a(e)$. In order to decouple the fields, notice that the operator

$$ E_{\mu\nu}(e,e) = \eta_{\mu\nu} + (e_\nu\partial_\mu + e_\mu\partial_\nu)I_e + e^2\partial_\mu\partial_\nu I_e^2 $$

acts in momentum space on the creation and annihilation parts by multiplication with $E(e,e)_{\mu\nu}(p)$ and with $E(e,e)_{\mu\nu}(-p) = E(-e,-e)_{\mu\nu}(p)$, respectively. Thus,

$$ A^{(2)}_{\mu\nu}(e) := A_{\mu\nu}(e) + \frac{1}{2}E_{\mu\nu}(e,e)\,a^{(0)}(e) \tag{3.7} $$

decouples from $a^{(0)}$, and its 2-point function is the same as Eq. (3.5) but with the proper coefficient $-\frac{1}{2}$ rather than $-\frac{1}{3}$ for the last term. Thus, $A^{(2)}$ is a string-localized potential for the massless field strength. It is, unlike other potentials, positive, traceless and conserved.

Now, the DVZ discontinuity can be replaced by a smooth limit of the massive potential $A^{(2)}$ to the massless $A^{(2)}$. The apparent "jump" of the coefficient from $-\frac{1}{3}$ (massive) to $-\frac{1}{2}$ (massless) (cf. also [22, Eq. (28)]) is by itself *not* a discontinuity, but a consequence of the choice of the fields before the limit is taken.

In the massless limit, $A^{(0)}(e) = \sqrt{3/2}\,a^{(0)}(e)$ becomes the e-independent massless scalar field by Eq. (3.6). $A^{(1)}_\mu := \sqrt{2}\,a^{(1)}_\mu$ is the same string-localized Maxwell potential as obtained from $s = 1$.

The generalization of Eq. (2.4) quantifies the singular lower helicity contributions to A^{P}:

$$\begin{aligned} A^{\mathrm{P}}_{\mu\nu}(x) \;&=\; A^{(2)}_{\mu\nu}(x,e) - \sqrt{1/6}\,E_{\mu\nu}(e,e)\,A^{(0)}(x,e) - \\ &-\frac{\sqrt{1/2}}{m}\left(\partial_\mu A^{(1)}_\nu + \partial_\nu A^{(1)}_\mu\right)(x,e) + \frac{\sqrt{2/3}}{m^2}\,\partial_\mu\partial_\nu A^{(0)}(x,e). \end{aligned}$$

Also the massless potential $A^{(2)}_{\mu\nu}(e)$ can be averaged over the string directions with $e^0 = 0$, and yields the Coulomb gauge potential $A^{\mathrm{C}}_{0\mu}(x) = 0$.

The case of general integer spin [12] is very similar to $s = 2$, except for the more involved combinatorics.

4 String-localized stress-energy tensor

The stress-energy tensor is by no means unique. It must be conserved and symmetric so that the generators

$$P_\sigma = \int_{x_0=t} d^3x\, T_{0\sigma}, \quad M_{\sigma\tau} = \int_{x_0=t} d^3x\,(x_\sigma T_{0\tau} - x_\tau T_{0\sigma})$$

are independent of the time t; and the commutators with the generators must implement the infinitesimal Poincaré transformations given by the Wigner representation. (The commutators are fixed by the 2-point functions.) But one may add "irrelevant" local terms as long as they do not change the generators.

The correct generators are obtained from the "reduced stress-energy tensor" ($\times = \mu_2 \dots \mu_r$ is a multi-index)

$$T^{\mathrm{red}}_{\rho\sigma} := (-1)^s\Big[-\frac{1}{4}{:}A^{\mathrm{P}}_{\mu\times}\,\overset{\leftrightarrow}{\partial}_\rho\overset{\leftrightarrow}{\partial}_\sigma\,A^{\mathrm{P}\mu\times}{:} - \frac{s}{2}\,\partial^\mu\Big({:}A^{\mathrm{P}}_{\rho\times}\,\overset{\leftrightarrow}{\partial}_\sigma\,A^{\mathrm{P}\times}_\mu{:} + (\rho\leftrightarrow\sigma)\Big)\Big]. \tag{4.1}$$

It differs by "irrelevant terms" from the Hilbert tensor, defined as the variation of a suitable generally covariant action w.r.t. the metric. The first term in Eq. (4.1) also appears in Fierz [3]. The second term does not contribute to the momenta, but is needed to ensure the correct Lorentz transformations [12].

Expanding $A^{\rm P}$ into $A^{(r)}(e)$ resp. $A^{(r)}(e')$, and discarding irrelevant terms (involving escort fields) that "carry away" all singularities when $m\to 0$, one gets a string-localized stress-energy tensor that admits a massless limit. Discarding more terms that are irrelevant at $m=0$, one decouples it as the sum over $r\leq s$ of

$$T^{(r)}_{\rho\sigma}(e,e') = (-1)^r\Big[-\frac{1}{4}{:}A^{(r)}_{\mu\times}(e)\,\overset{\leftrightarrow}{\partial}_\rho\overset{\leftrightarrow}{\partial}_\sigma\,A^{(r)\mu\times}(e'){:} - \frac{r}{4}\,\partial^\mu\Big({:}A^{(r)}_{\rho\times}(e)\,\overset{\leftrightarrow}{\partial}_\sigma\,A^{(r)\times}_\mu(e'){:}\ \begin{matrix}+(e\leftrightarrow e')\\ +(\rho\leftrightarrow\sigma)\end{matrix}\Big)\Big] \quad (4.2)$$

understood as distributions in two independent directions e,e'. As in Eq. (3.3),

$$A^{(r)}_{\mu_1\dots\mu_r}(x,e) = (I^r_e F^{(r)}_{[\mu_1\nu_1]\dots[\mu_r\nu_r]})(x)\,e^{\nu_1}\dots e^{\nu_r}$$

can be expressed in terms of the massless field strengths.

As the massless potentials $A^{(r)}$ mutually commute, the generators defined by $T^{(r)}$ implement the Poincaré transformations of $A^{(r)}$. Massless higher spin currents are constructed similarly. For details see [12].

That the Weinberg-Witten theorem can be evaded with non-local densities, was pointed out earlier in [9], where examples with unpaired helicities were given. Eq. (4.2) involving string integrals over field strengths is perhaps the most conservative alternative, also in comparision with other proposals to couple higher spin matter to gravity [5, 23, 1].

5 "Fattening"

The 2-point functions of the massless and massive string-localized potentials $A^{(s)}$ (for any spin) are the same polynomial in the tensor $E_{\mu\nu}(p)$, except that the argument p of the functions $E_{\mu\nu}$ is taken on the respective mass-shell. One obtains the massive field $A^{(s)}$ from the massless field just by changing the dispersion relation $p^0=\omega_m(\vec p)$. As the massive 2-point function was constructed on the Hilbert space of the Proca potential, this deformation preserves positivity. Through the coupling to the lower escort fields, it brings back all spin components of the Proca field. Indeed, the latter is restored from the massive potential $A^{(s)}$ by

$$A^{\rm P}_{\mu_1\dots\mu_s}(x) = (-1)^s{}_mM^{A^{\rm P}_{\mu_1\dots\mu_s},A^{{\rm P}\nu_1\dots\nu_s}}\,A^{(s)}_{\nu_1\dots\nu_s}\big|_m(x,e),$$

where ${}_mM^{A^{\rm P},A^{\rm P}}$ is understood as a differential operator ($\pi_{\mu\nu}=\eta_{\mu\nu}+m^{-2}\partial_\mu\partial_\nu$).

6 Conclusion

We have identified string-localized potentials for massive particles of integer spin s on the Hilbert space of their field strengths, that admit a smooth massless limit to decoupled potentials with helicities $h=\pm r$, $r\leq s$. We have presented an inverse "fattening" prescription via a manifestly positive deformation of the 2-point function. The approach provides a way around the Weinberg-Witten theorem, and sheds new light on the DVZ discontinuity.

Our results also allow to approximate string-localized fields in the massless infinite-spin Wigner representations [15] by the massive scalar escort fields $A^{(0)}$ of spin $s \to \infty$, $m^2 s(s+1) = \kappa^2 = \text{const.}$ (Work in progress [13].)

String-localized fields are a device to formulate quantum interactions in terms of a given particle content. Their renormalized perturbation theory is presently investigated [7, 14, 10]. It bears formal analogies with BRST renormalization, but is more economic (avoiding unphysical degrees of freedom), and much closer to the fundamental principles of relativistic quantum field theory.

It was shown in the framework of algebraic quantum field theory, that to connect scattering states with the vacuum, may in certain theories require operations localized in narrow spacelike cones; and in the presence of a mass gap it cannot be worse than that [2]. The emerging perturbation theory using string-localized fields is the practical realization of this insight.

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