Beyond gauge theory: Hilbert space positivity and causal localization in the presence of vector mesons

Dedicated to the memory of Raymond Stora and John Roberts

Bert Schroer
present address: CBPF, Rua Dr. Xavier Sigaud 150,
22290-180 Rio de Janeiro, Brazil
permanent address: Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany

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Abstract

The Hilbert space formulation of interacting $s=1$ vector-potentials stands in an interesting contrast with the point-local Krein space setting.of gauge theory. Already in the absence of interactions the Wilson loop in a Hilbert space setting has a "topological property" which is missing in the gauge theoretic description (Haag duality, Aharonov-Bohm effect); the conceptual differences increase in the presence of interactions.

The Hilbert space positivity weakens the causal localization properties if interacting fields which results in the replacement of the gauge-variant point-local matter fields in Krein space by string-local physical fields in Hilbert space. The gauge invariance of the perturbative S-matrix corresponds to its independence of the spacelike string direction of its interpolating.fields. In contrast to gauge theory, whose physical range is limited to gauge invariant perturbative S-matrix and local observables, its Hilbert space string-local counterpart in is a full-fledged quantum field theory (QFT).

The new setting reveals that the Lie-structure of self-coupled vector mesons results from perturbative implementation of the causal localization principles of QFT.

1 Introduction

It is well known that the use of point-local massless vector potentials is incompatible with the positivity of Hilbert space. One usually resolves this problem by abandoning positivity and maintaining the point-local field formalism which leads to gauge theory (GT). The prize to pay is well-known from quantum electrodynamics (QED) in the standard indefinite metric (Gupta-Bleuler) gauge setting: positivity can be recovered for local observables, whereas charge-carrying fields remain outside its physical range. The separation between physical and unphysical quantum fields is done in terms of gauge symmetry which is not a physical symmetry but a formal device to extract a physical subtheory.

Although the standard gauge formalism is restricted to vector potentials, the clash between zero mass point-local fields and positivity is a general phenomenon for all $s \geq 1$ zero mass tensor potentials. It does not affect the corresponding field strengths, but the short distance dimension of the latter ($d_{sd} = 2$

for $s = 1$) would prevent their use in renormalizable interactions. For semi-integer spin the borderline is $s \geq 3/2$; such fields will play no role in this paper.

Another possibility is to keep the Hilbert space, but allow a weaker localization. The tightest covariant localization of fields beyond point-like is causal localization on semi-infinite spacelike "strings"[1] $x + \mathbb{R}_+ e,\ e^2 = -1$. It is easy to construct $m > 0$ covariant fields $\Psi(x, e)$ in terms of semi-infinite line integrals. Whereas the point-local (pl) Proca vector-potentials have no massless limit, the correlation functions of their string-local (sl) siblings pass without problems to their massless counterpart. The construction of free massive sl potentials in the same Hilbert space as their point-local counterparts guaranties that both belong to the same relative localization class. They represent two different "field coordinatizations" of the same theory, which (in the presence of interactions) implies that their particle content and S-matrix are identical.

The perturbative use of string-local elementary matter fields $\Psi(x, e)$ in Hilbert space quantum field theory[2] (QFT) correspond to formally gauge invariant string-local composites in terms of pl fields in GT

$$\psi^K(x)e^{ig\int_0^\infty A_\mu^K(x+\lambda e)e^\mu d\lambda} \tag{1}$$

where the superscript K refers to the pl indefinite metric (Krein space instead of Hilbert space) setting of GT. Proposals to recover gauge-invariant charge-carrying matter fields in terms of such formal expressions appeared already a long time before renormalization theory [1]. They played no role in the discovery of renormalized perturbation theory, but they re-appeared later in Mandelstam's proposal to replace the perturbative gauge theoretic setting of QED by one which uses solely Hilbert space gauge-invariant field strengths instead of Krein space vector potentials [2]. Attempts to obtain mathematical control of such non-linear and non-local composites as (1) within renormalized perturbation theory remained without success.

The present paper solves this old problem by using massive string-local vector potentials which are defined as semi-infinite spacelike line-integrals over field strengths. They are covariant fields which maintain the Hilbert space positivity and in contrast to the Proca potentials permit, as aforementioned, smooth $m \to 0$ limits of their vacuum expectation values. Their use opens new areas of exploration which are outside the physical range of gauge theory.

In the new setting the lowest order perturbative interaction densities are defined in terms of Wick products of string-local covariant free vector potentials $A_\mu(x, e)$ (obtained by integrating point-local field strengths along spacelike semi-lines) with point-local free $s < 1$ matter fields. The higher order interactions spread the string-localization of the potentials to the $s < 1$ matter fields so that their zero order pl fields turn into higher order (in a certain sense even stronger sl than the potentials) sl $\Psi(x, e)$ fields. These sl matter fields are the elementary interacting matter fields of perturbation theory which replace the composite expressions in the Krein fields (1).

The problem is then to extend renormalization theory from pl to sl fields. The feasibility of such a project relies on the observation that sl fields have a better short distance behavior than their pl counterparts. Whereas the short distance dimension d_{sd} of pl free field tensor-potentials increases with spin as $d_{sd}^s = s + 1$, that of their string-local siblings remains at $d_{sd}^s = 1$ *independent of* s. This implies that there is no problems to construct interaction densities *for any spin within the power-counting bound (PCB)* $d_{sd}^{int} \leq 4$ of renormalizability. In the present paper the spin of the $s \geq 1$ higher spin fields will be integer; for spinor fields there are corresponding results.

However in contrast to interactions between $s < 1$ pl fields, for which the first order PCB is the only restriction, there is an additional requirement for sl fields which has no pl counterpart and bears no

[1] Beware that strings in String Theory are not string-local in the sense of local quantum physics.

[2] Unless stated otherwise, QFT refers to the Hilbert space setting of quantum theory. Gauge theory, which contains a Hilbert space subtheory, will be referred to as GT.

relation to short distance properties. Higher order terms will in general not remain sl but rather lead to a *total delocalization*; such situations turn out to be inconsistent with the principles of QFT. Heuristically this new phenomenon can be understood in terms of the x-integration in inner propagator lines which, in case of x being the starting point of a semi-infinite spacelike line $x+\mathbb{R}_+e$ converts the string-localization into a complete delocalization. The avoidance of this total delocalization imposes a severe conditions on the interaction density. In contrast to the PCB short distance renormalizability requirement the new restriction maintains the sl of those fields whose vacuum expectation values one wants to computes but prevents the dependence on "inner" $e's$. The maintenance of string-localization is important for the physical long-distance properties.

It vaguely corresponds to the requirement of gauge invariance. But whereas the latter bears no direct relation to the foundational principles of QFT (The pl localization of gauge-dependent fields in a Krein space is not the physical localization !), the prevention of total delocalization is directly related to the causal localization principles of QFT. Whereas the Krein space asymptotic short distance properties probably agree with those in the sl Hilbert space setting[3], the understanding of long distance properties requires the use of sl fields. It is well-known that in a Hilbert space setting there are no renormalizable interaction densities involving pl $s \geq 1$ fields.

Some of these nonrenormalizable pl interactions turn out to be renormalizable in terms of $s \geq 1$ covariant sl fields. This is in particular the case if one allows higher order short distance compensations between couplings containing fields of different spins (reminiscent of compensations between different spin components in supersymmetry multiplets). Those which are not renormalizable in the sl sense are in all likelihood not models of QFT. This points to an interesting connection between perturbative renormalizability and causal localization properties. The main purpose of this paper is to illustrate these new ideas in their simplest possible low order perturbative context. Technically more demanding tasks, as the elaboration of an n^{th} order Epstein-Glaser renormalization theory which includes causal string-crossings, will be left to future publications.

Quantum gauge theory also achieves a reduction from $d_{sd} = 2$ to $\dot{d}_{sd} = 1$ in case of $s = 1$, but it accomplishes this not with a Hilbert space positivity maintaining weakening of localization from pl to sl, but rather with the help of indefinite metric which permits compensations in intermediate states between positive and negative contributions by brute force. If it were not be for the existence of a subset of gauge invariant operators (which includes the S-matrix), an indefinite metric setting would remain without physical content.

The Hilbert space setting of QT is the basis of its probabilistic interpretation. It has no counterpart in the classical theory and hence one cannot rely on "quantization" to solve conceptual problems of interactions involving $s = 1$ fields. Quantum gauge theory is the result of keeping pl $s \geq 1$ fields at the prize of sacrificing positivity. Gauge symmetry is a local symmetry in classical electromagnetism; it bears no relation to the principles of QFT but rather enters QFT through the "quantization" parallelism to classical field theory[4]. Perturbative QFT needs no reference to Lagrangian quantization; it can be defined in terms of interaction densities formed from covariant pl or sl free fields which can be directly obtained from Wigner's positive energy representation theory of the Poincaré group.

Perturbative gauge invariance is a way to extend the pl formalism to $s = 1$ interactions in a Krein space in order to extract those quantities which do not "feel" the presence of negative metric states (namely the local observables and the S-matrix). It has been "an amazingly successful placeholder" (Stora) for an unknown Hilbert space formulation within the standard setting of perturbative QFT. The new sl Hilbert space setting extends the Hilbert space Wightman formulation for interacting $s < 1$ pl fields to $s \geq 1$ sl

[3]This would be necessary in order to secure the asymptotic freedom as a property which is consistent with positivity of QFT..

[4]There are however classical limits of pl or sl free quantum fields in terms of expectation values in coherent states.

fields for which the test-function smearing amounts to smearing in x, using the smooth rapidly deceasing test functions $f(x)$) *and* smearing in e, using smooth compact supported functions $h(e)$ in the $d = 1 + 2$ de Sitter space of spacelike directions e, $e^2 = -1$[5]. The main purpose of the present work is to convince the reader that the beginnings of a theory for which gauge theory is the placeholder already exist.

The idea of constructing string-localized fields in Hilbert space can be traced back to the solution of the localization properties of Wigner's zero mass infinite spin representation class [3] [4] in terms of the *modular localization theory* of algebraic QFT (a historic/philosophical view can be found in [5]). But, as it is often the case, the historic context which led to a new view of QFT may turn out to be less important for its use in actual problems; the construction of string-local fields and their use in a new perturbative setting does not require knowledge about modular localization.

A prior result from algebraic QFT (AQFT) which highlighted the *naturalness of string-localization* was obtained by Buchholz and Fredenhagen in [6]. Their theorem was formulated and proven in the operator-algebraic setting; re-expressing its physical content in terms of quantum fields it states:

Theorem: *The field theoretic content of an asymptotically complete QFT with a mass gap and local observables can be fully accounted for in terms of string-local covariant fields $\Psi(x,e)$ localized on spacelike semi-lines $x + R_+e$ $e^2 = -1$. Point-local observable fields correspond to the special case of e-independent $\Psi's$. QFT does not need fields with weaker than stringlike localization properties (as e.g. localization on spacelike hypersurfaces).*

The Hilbert space of such a theory can be described as a Wigner-Fock Hilbert space spanned by those particle states which the (LSZ or Haag-Ruelle) scattering theory associates asymptotically with the string-local fields. Since functional analytic and operator-algebraic tools are not available in indefinite metric Krein spaces, the gauge theoretic setting is strictly limited to the combinatorial rules of finite order renormalized perturbation theory whereas nonperturbative theorems (TCP, Spin&Statistics, scattering theory,...) cannot be derived. On the other hand most mathematical tools of (Wightman) QFT remain valid for string-local fields; hence large parts of textbook presentation of nonperturbative QFT can be adjusted to the new situation.

More specific results come from the new perturbation theory of sl fields. Whereas positivity-preserving renormalizable interactions between point-local fields are limited to low spins $s < 1$, renormalizable interactions containing $s \geq 1$ fields in a Hilbert space setting require a weakening of localization from point- to string-local fields [7] [8]. Couplings which even remain nonrenormalizable in a string-local setting are not expected to represent models of QFT.

The a priory knowledge that the Hilbert space in models with a mass gap can be identified with a Wigner-Fock particle space turns out to be very useful in the perturbative construction[6]. This well understood situation is also a good starting point for investigating massless limits in which the standard Wigner particle structure is lost (which manifests itself in terms of infrared divergencies) and the appropriate Hilbert space has to be constructed from the massless limit of the massive correlation functions [12]. Most deep unsolved problems of QFT, in particular a spacetime understanding of infrared properties which account for the conceptual changes of large time scattering properties in QED and are responsible for confinement in QCD, are connected with interactions involving massless vector potentials. Already for free fields the massless field is most conveniently constructed through the massless limit of the free massive sl correlation functions.

In view of the strong relation between causal localization and Hilbert space positivity one cannot expect that GT accounts for the correct causal localization properties. The absence of of positivity (unitarity)

[5]The present experience with perturbation theory suggests that interacting Wightman fields with $s \geq 1$ exist only in the form of sl fields.

[6]It is important that this Fock spaces refers to asymptotic fields (particles); the use of a Fock space at finite times is forbidden by "Haag's theorem" [13].

is not only affecting issues of Hermitian adjoints and unitarity, but also spoils the foundational causality property of QFT. The positivity and the ensuing string-localization is particularly important for large distance properties which usually announce their presence through infrared divergencies. Long distance problems do not occur in $s < 1$ interactions; their physical understanding is inexorably related to properties of $s \geq 1$ interacting sl fields. Although not yet established, one expects that the short distance asymptotic freedom behavior of the gauge dependent matter fields will be confirmed in the sl Hilbert space theory.

The role of sl fields will be illustrated in two problems. One problem, the so-called "violation of Haag duality", is closely related to the Aharonov-Bohm effect. It shows that the use of Hilbert space string-local vector potentials in a Wilson loop leads to a topological effect which is absent for gauge theoretic point-like potentials in Krein space and has generalizations to all massless $s \geq 1$ fields. The somewhat "eery feelings" about an apparent causality violation (which partially account for the popularity of the A-B effect) can be traced back to the use of the point-local potentials of gauge theory; they disappear once one uses instead their covariant string-local counter-part which perceive the difference between Haag suality and the more general Einstein causality.

The main problem posed by string-local fields is the construction of the correct first order interaction densities. It is well-known that the requirement of PCB $d_{sd}^{int} \leq 4$ (the prerequisite for renormalizability) only permits pl couplings between fields of low spin $s < 1$ with $d_{sd} = 1$ or $3/2$ (for $s = 1/2$). String-local fields, whose short distance dimension is $d_{sd} = 1$ for bosons (and $3/2$ for fermions *independent of* s), permit PCB couplings for any spin.

But, as already mentioned before, there exists an additional rather severe restriction on sl interaction-densities which has no counterpart for renormalizable interactions between $s < 1$ pl fields. Most string-local interactions within $d_{sd}^{int} \leq 4$ lead to a *total delocalization in higher orders*, which removes them from the list of potential candidates for a QFT model. This restriction will be exemplified for three models: couplings of massive string-local vector mesons to complex fields (massive QED), to Hermitian fields (the Higgs model) and self-coupled massive vector mesons.

The presence of self-coupled massive vector mesons lead to a new phenomenon; it is the only known coupling for which the first order PCB does not imply renormalizability; or to formulate it in a historical context: passing from the 4-Fermi interaction to intermediate self-interacting massive vector mesons improves the short distance properties so that the coupling leads to first order PCB, but fails in second order. The culprit is a second order "induced" $d_{sd} = 5$ self-coupling which can only be compensated by enlarging the first order coupling by a $A \cdot AH$ coupling with a scalar Hermitian H field whose second order iteration contains a compensating H-independent term. Whereas the massive (scalar, spinor) vector mesons of QED do not require the presence of a (or several) H, the second order PCB of string-local massive Y-M and QCD cannot be maintained without a compensating H-coupling.

The combination of a shift in the complex field space of a massless scalar QED (SSB "breaking of gauge symmetry") and a subsequent reality adjustment by an operator gauge transformation (no counterpart in SSB) leads also to the correct interaction density, but this "Higgs mechanism" bears no physical relation with the intrinsic origin of the second order induced (not postulated !) Mexican hat potential of a H self-interaction (section 4). A genuine physical SSB breaking on the other hand leaves its intrinsic (i.e. independent of by what prescription it was obtained) physical mark on the final model and there is simply none. The identically conserved Maxwell current $j_\mu = \partial^\nu F_{\mu\nu}$ of an abelian massive vector meson leads to a screened (and not to spontaneously broken) charge [8]. Interactions between Hermitian fields and massive vector mesons are (as their complex counterparts) fully accounted for by the BRST gauge invariance of the S-matrix $\mathfrak{s}S = 0$ or its spacetime Hilbert space counterpart namely the independence of S on string directions $d_e S = 0$.

The perturbative string-local field theory (SLFT) shares many *formal* similarities with gauge theory (GT). This is particularly evident if one formulates GT not in terms of Feynman rules but instead uses the so-called causal gauge invariant operator setting (CGI) [9]. The reason is that gauge invariance must hold on-shell but is violated off-shell. This makes it difficult to discuss properties as positivity (unitarity), whose validity in gauge theories is *limited to the on-shell restrictions* of correlation functions (i.e. the S-matrix). In the CGI operator formulation of the BRST gauge formalism [9] [10] [11] the perturbative implementation is focussed on the construction on the S-operator which fulfills $\mathfrak{s}S = 0$ where $\mathfrak{s}$ is the nilpotent BRST s-operation; in this way the construction of the gauge-invariant S-operator is separated from the construction of the less interesting gauge dependent correlation functions.

The implementation of the string-independence of the S-matrix and the independence of "inner" strings of correlation functions of string-local fields is achieved with the help of a differential form calculus on the $d = 1+2$ de Sitter space of string directions. Unlike GT with its cohomological BRST formalism based on a nilpotent $\mathfrak{s}$-operation, whose physical range is restricted to local observables and the perturbative S-matrix, the SLFT is a full QFT which contains the important string-local physical matter fields whose LSZ limits connect the causal localization principles implemented in terms of fields and their charge-carrying local equivalence classes with the observable world of individual particles just as in point-local Hilbert space QFTs. But the appearance of the e-dependence in going from on- to off-shell complicates the analytic connection between scattering amplitudes and correlation functions.

It is interesting to use some hindsight from this new developments for looking back at the origin of modern GT. In the 't Hooft-Veltman paper on the renormalization of nonabelian gauge theory the important issue was the verification of the unitarity of the S-matrix (ignoring infrared problems). What they achieved with the help of lengthy "by hand" calculations was afterwards "streamlined" by adding formal tools. Starting with Faddeev-Popov, being enriched by Slavnov, the formal polishing finally reached its present perfection in the hands of Becchi, Rouet, Stora and Tuytin. It would be impossible for scholars of QFT to derive the BRST gauge formalism from the principles of QFT; they remain useful inventions for extracting a physical subtheory (in particular the S-matrix) from an unphysical description in Krein space.

The SLFT Hilbert space setting contains only physical degrees of freedom. But this conceptual economy leads to new and even somewhat surprising concepts. The massive pl Proca field $A_\mu^P(x)$ turns out to lead to a *pair* of sl fields, a sl vector potential $A_\mu(x,e)$ *and* an sl scalar field $\phi(x,e)$. Both fields appear in the interaction, but in the zero mass limit only the sl A_μ survives.This has a surprising analogy with the appearance of short range vector potentials in the BCS theory of superconductivity. In this analogy the bosonic ϕ field corresponds to the Cooper pairs. As the ϕ field, the Cooper pairs result from a reorganization of existing degrees of freedom.

Neither in the many-body quantum mechanical description of BCS superconductivity which leads to London's short range vector potentials, nor for the description of QFT interactions of massive abelian vector mesons with matter one needs new (e.g. Higgs) degrees of freedom. The QFT counterpart of the Cooper pair regrouping are the ϕ fields; it will be shown that they are an indispensable epiphenomenon of interacting massive vector mesons in a Hilbert space. This clears the way for asking the question why one needs H-fields in the presence of self-interacting massive vector mesons. The answer has already been given in [8] and will receive additional attention in section 4.

Whereas the SLFT Hilbert space formalism has many formal similarities with the implementation of on-shell BRST gauge invariance (section 5) in the operator formulation of "causal gauge invariance" (CGI) of the University of Zürich group published during the 90s [9] [10], its conceptual and calculational power reaches beyond when it comes to long distance infrared problems which cannot be described in terms of an S-matrix. The correct analog of the long-range Coulomb interaction of quantum mechanics are the long distance properties of physical matter fields interacting with massless sl vector potentials. In such cases the structure of a Wigner-Fock Hilbert space and an S-matrix acting in it breaks down and the remaining objects from which the theory has to be reconstructed are the massless limits of the sl correlation functions.

The next section presents the kinematics of SLFT i.e. relations between the string-local free fields and their use in interaction densities for which one does not need perturbative calculations. The third section addresses what the title of this paper promises. Some second order perturbative results and their interpretation are presented in section 4; this section also contains educated guesses about physical manifestations of infrared properties. Section 5 presents formal analogies with the CGI operator formulation of the BRST formalism. The outlook contains comments about ongoing calculations and conjectures about what one hopes to accomplish in the future.

Many of the ideas arose in extensive discussions over several years with Jens Mund, but the responsibility for possible errors and less than perfect presentations rest on my shoulders.

2 A Hilbert space alternative to local gauge theory, kinematical aspects

Massive free spin $s \geq 1$ fields are commonly described in terms of degree s tensor potentials. For $s = 1$ this reduces to the well-known Proca potential

$$A_\mu^P(x) = \frac{1}{(2\pi)^{3/2}} \int (e^{ipx} \sum_{s_3=-1}^{1} u_\mu(p, s_3) a^*(p, s_3) + h.c.) \frac{d^3p}{2p_0} \tag{2}$$

$$\langle A_\mu^P(x) A_\nu^P(x') \rangle = \frac{1}{(2\pi)^3} \int e^{-ip\xi} M_{\mu\nu}(p) \frac{d^3p}{2p_0},\ M_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

To

To this point-local field one may associate two string-local fields, a vector potential $A_\mu(x,e)$ and a scalar field $\phi(x,e)$ [4], as well as two "field-valued differential forms" in e-space namely a one-form $u(x,e)$ and a two-form $\hat{u}(x,e)$

$$A_\mu(x,e) = \int_0^\infty F_{\mu\nu}(x + \lambda e) e^\nu d\lambda,\ with\ F_{\mu\nu}(x) = \partial_\mu A_\nu^P(x) - \partial_\nu A_\mu^P(x), \tag{3}$$

$$\phi(x,e) = \int_0^\infty A_\mu^P(x + \lambda e) e^\mu d\lambda, \quad e^2 = -1$$

$$u = d_e \phi = \partial_{e^\mu} \phi de^\mu,\ \hat{u} = d_e(A_\alpha de^\alpha)$$

which are all members of the equivalence class of relatively string-local fields which are associated to the point-local A_μ^P Proca field (the sl "Borchers class"). It is well known [12] that in the presence of a mass gap interacting point-local fields within the same localization class lead to the same particle physics (particles, S-matrix); this continues to be valid for string-local fields [6] [13].

Throughout this paper the differential form calculus on the $d = 1+2$ de Sitter space of string-directions will play an important role. As the x, the $e's$ are variables in which the fields fluctuate; they bear no relation with the "mute" gauge-fixing parameters of GT.

String-local vector potentials fulfill $e^\mu A_\mu(x,e) = 0$, and in the massless limit also $\partial^\mu A_\mu(x,e) = 0$. These relations are not imposed gauge conditions but rather intrinsic properties of string-local potentials which result from the above definitions. Note that A_μ, u and $\hat{u}$ possess zero mass limits, whereas A_μ^P and ϕ remain finite only in the combination $A^P + \partial\phi$. They are all free fields, but their mutual dependence leads to mixed 2-point functions (below).

One may change the string "density" $d\lambda \to q(\lambda)d\lambda,\ q(\infty) = 1$ within the linear field class. There is no conceptual problem with this continuous enlargement since quantum fields, in contrast to classical fields, have no observable "individuality". The latter property is an attribute of particles which share their superselected charges with their associated field-classes with which they are (large-time) asymptotically related [13]. Renormalizability requires to define PCB interaction densities in terms of maximally fourth degree polynomials of *fields with the lowest short distance dimension within a field class* but the distinction between elementary and composite is not based on the short distance dimensions of fields but pertains to the fusion properties of superselected charges of field classes.

In order to obtain linear relations between these fields as

$$A_\mu = A_\mu^P + \partial_\mu \phi \tag{4}$$

one must use the same $q(\lambda)$ and in order to maintain simplicity of two-point function

$$\langle A_\mu(x,e) A_{\mu'}(x',e') \rangle = \frac{1}{(2\pi)^3} \int e^{-ip(x-x')} M^{A,A}_{\mu\mu'}(p;e,e') \frac{d^3p}{2p_0} \tag{5}$$

$$M^{A,A}_{\mu\mu'}(p;e,e') = -g_{\mu\mu'} - \frac{p_\mu p_{\mu'}(e\cdot e')}{(p\cdot e - i\varepsilon)(p\cdot e' + i\varepsilon)} + \frac{p_\mu e_{\mu'}}{(p\cdot e - i\varepsilon)} + \frac{p_\mu e'_{\mu'}}{(p\cdot e' + i\varepsilon)}$$

$$M^{\phi\phi} = \frac{1}{m^2} - \frac{e\cdot e'}{(p\cdot e - i\varepsilon)(p\cdot e' + i\varepsilon)},\ M^{A\phi}_\mu = \frac{1}{i}\left(\frac{e'_\mu}{p\cdot e' + i\varepsilon} - \frac{p_\mu e\cdot e'}{(p\cdot e - i\varepsilon)(p\cdot e' + i\varepsilon)}\right)$$

These expressions may be either directly derived from the above line integral or be obtained from the A, A^P and ϕ-intertwiners[7] (the e-dependent denominators including the ε-prescription result from the Fourier transformation of the Heavyside function). Clearly the λ-integration (3) has lowered the short distance dimension of the vector potential from $d_{sd}^P = 2$ to $d_{sd} = 1$ and the heuristic reading of (4) is that the derivative $\partial\phi$ of the $d_{sd} = 1$ ϕ-escort removes the most singular part of A^P at the price of directional $e-$fluctuation. In the massless limit the A^P as well as ϕ diverge, but the string-local potential $A_\mu(x,e)$ remains well-defined; its 4-dimensional curl is the point-local field strength.

The necessity to work with string-local potentials and the *appearance of mixed two point functions* is the prize for working in the *Hilbert space of physical degrees of freedom.* In contrast the indefinite metric Krein space of gauge theory contains in addition to the Gupta-Bleuler and Stückelberg indefinite metric degrees of freedom also contains those of "ghost" fields. There are no mixed contributions since these unphysical free fields are independent. The BRST gauge formalism permits to extract physical data but falls short of extracting a full QFT.

The sl $A_\mu(x,s)$ and its scalar escort $\phi(x,e)$ are different fields, but they do not enlarge the degrees of freedom; in fact they are linear in the same Wigner creation and annihilation operators $a^\#(p,s_3)$ of the point-local Proca potential. Although they do not change the degrees of freedom, their separate appearance in interaction densities plays an important role in upholding string localization of higher order interacting fields. The QFT of the textbooks, in which each particle corresponds to one pl field, is limited to interactions between $s < 1$ fields. QFT of $s \geq 1$ requires the use of sl fields and each such field is accompanied by s lower spin sl escorts (see later).

In order to prevent confusion, the present work uses the terminology "QFT" only for genuine quantum theories in Hilbert space. Gauge theories in Krein space are not QTs as they stand (since the positivity is not less important than $\hbar$), but the BRST formalism permits to extract quantum subtheories (local observables acting in the Hilbert space vacuum sector, the S-matrix acting in a Wigner-Fock particle space). The nonperturbative mathematical tools which lead to the famous theorems of local quantum physics (TCP, Spin&Statistics,...) require the presence of positivity (unitarity).

[7]A systematic and detailed account of the construction of string-local fields and their use in a string-extended Epstein-Glaser construction of time-ordered products will be contained in a forthcomong manuscript by Jens Mund..

The reader may encounter many new concepts, but he can be assured that they are not ideosyncratic inventions of the author but rather result from reconciling higher spin interactions with the causal localization principles in the Hilbert space setting of QT. In particular the existence of massless vector- (more general $s > 1$ tensor-) potentials in Hilbert space is tied to the existence of the $m \to 0$ limit of sl massive correlation functions. These massless sl higher spin fields are the counterparts of the pl massless $s < 1$ fields; their use is indispensable for the understanding of the spacetime physics behind logarithmic infrared divergencies.

The above relation (4) resembles a gauge transformation. It should be maintained in the presence of interactions with a complex matter field. Whereas the interaction density is defined in terms of a free pl matter field $\psi_0(x)$, the interaction with sl potentials will convert these fields into an interacting sl fields $\psi(x,e)$. One also expects that these sl field has a very singular interacting pl sibling ψ^P (the analog of the pl fields in the nonrenormalizable pl Hilbert space setting) and that both are related (in the sense of normal products) as[8]

$$\psi(x,e) = N\psi^P(x)e^{ig\phi(x,e)} \tag{6}$$

This together with (4) is certainly reminiscent of a gauge transformation of a matter field interacting with a vector potential. But in the present context it represents a relation between two "field coordinatizations" of the same theory, one being a string-local Wightman field (bounded d_{sd}) and the other a field with unbounded short distance dimension as expected in nonrenormalizable couplings [14]. This is a class of fields which are too singular (unbounded d_{sb}) in order to be compactly localizable in the sense of Wightman [12]; such fields have been studied by Jaffe [15] who illustrated his more singular fields in terms of Wick-ordered exponentials of free fields $\exp\varphi$.

After this brief excursion into uncharted territory, this section returns to kinematic aspext of sl free fields.

The formal similarity of the directional variable e of string-local fields with a (noncovariant) "axial" gauge parameter should not hide the fact that its gauge theoretic interpretation[9] caused unsolvable short- and long- distance problems, which finally led to its abandonment. The reason behind this failure is that fluctuations in the d=1+2 de unit Sitter space of spacelike directions in individual string-local fields cannot be reconciled with a gauge interpretation; fortunately *what was a curse in the use as an axial gauge turns out to be a blessing in the Hilbert space setting* of $s = 1$ interactions.

This construction permits a generalization to any integer spin. Massive free fields of spin s and short distance dimension $d_{sd}^s = s + 1$ are conveniently described in terms of symmetric point-local potentials $A^P_{\mu_1..\mu_s}$ of tensor degree s. A corresponding string-local tensor with $d_{sd}^s = 1$ can be obtained in analogy to the vector potential with the help of repeated semi-infinite line integrals (see appendix)

$$\phi_{\mu_1,..\mu_k}(x.e) = \int_0^\infty .. \int_0^\infty d\lambda_{k+1}..d\lambda_s e^{\mu_{k+1}}..e^{\mu_s} A^P_{\mu_1,\mu_2,..,\mu_k,..,\mu_s}(x + \lambda_{k+1}e + ..\lambda_s e) \tag{7}$$

For the sake of simplicity of notation we specialize to $s = 2$ in which case the corresponding relation to (4) is

$$g_{\mu_1\mu_2}(x,e) = g^P_{\mu_1\mu_2}(x) + sym\ \partial_{\mu_1}\phi_{\mu_2}(x,e) + \partial_{\mu_1}\partial_{\mu_2}\phi(x,e) \tag{8}$$

where our notation pays tribute to the fact that the metric tensor of general relativity is the principle physical candidate for a symmetric second degree tensor. Note that in this case the string-local field has 2 string-local escorts, a scalar $\phi(x,e)$ and a vector $\phi_\mu(x,e)$. By construction the symmetric tensor fulfills $e^{\mu_1}g_{\mu_1\mu_2} = 0$

[8] I am indebted to Jens Mund for informing me that this relation has been checked in lowest nontrivial order.

[9] The gauge theoretic e is considered to be "mute" i.e. it is the same for all gauge fields and remains unaffected by Lorentz transformations.

The field strength associated to the symmetric $g_{\mu\nu}$tensor field of $g_{\mu\nu}$ $s=2$ field of degree 4, which has the same mixed symmetric-antisymmetric permutation symmetry as the Riemann tensor of relativity (the "linearized Riemann tensor") is

$$R_{\mu\nu\kappa\lambda}(x)=\frac{1}{2}antisym\ \partial_{\mu}\partial_{\kappa}g_{\upsilon\lambda} \tag{9}$$

This R-tensor, which is the $s=2$ analog of the the $s=1$ field strength $F_{\mu\nu}$, is the lowest degree point-local massless $s=2$ field.

The extension of (4) and (8) to spin $s>2$ should be clear: a massive point-local degree s tensor potential corresponds to a string-local potential of the same tensor degree and "ϕ escorts" of lower tensor degree of which only the string-local degree s potential has a massless limit. The lowest degree point-local tensor field which permits a massless limit is a field strength of degree $2s$ and short distance dimension $d_{sd}=2s$. In analogy to (9), results from the application of s derivatives and subsequent anti-symmetrization.

The intertwiner $u(p,s)$ for massive point-local tensor fields of degree s and short distance dimension $d_{sd}=s{+}1$ relate the $2s{+}1$ spin space with the space of symmetric covariant tensor. They are divergence free (as A_{μ}^{P}) and traceless. It is simpler to calculate their momentum space two point functions which consists of linear combinations of tensors of degree $2s$ formed from the Minkowski spacetime $g_{\mu\nu}$ and products of p_{μ} made dimensionless by multiplication with appropriate inverse mass powers. The requirement of vanishing trace and divergence determines the two-point function up to a numerical factor.

The differential geometric structure of the $d=1+2$ directional de Sitter space impart these string-local fields and related field valued differential forms with a rich differential geometric structure which plays an important role in the new positivity-maintaining SLF perturbation theory. In this setting all fields are physical; the gauge invariant local observables correspond to point-local (generally composite) fields, whereas the interacting matter fields $\Psi(x,e)$ are string-local generalizations of Wightman fields (polynomially bounded in momentum space or equivalently $d_{sd}<\infty$).

A surprising collateral kinematic result of this observation is the (easy to verify) statement that the angular averaging over e within a spacelike plane leads to the Coulomb (or radiation) vector potential. It is well-known that it acts in a Hilbert space but that its lack of covariance makes it unsuitable for renormalized perturbation theory. The fact that it results from directional averaging of covariant string-local potential (which plays the central role in the new covariant SLFT Hilbert space setting) may come as a surprise to some.

Before taking up the issue of interactions, it is interesting to compare the SLFT setting with the BRST gauge formalism. The latter is based on a the action of a nilpotent $\mathfrak{s}$-operation on indefinite metric fields in a Krein space ($K=$Krein) extended by "ghost operators". In the notation of [9] it reads

$$\mathfrak{s}A_{\mu}^{K}=\partial_{\mu}u^{K},\ \mathfrak{s}\phi^{K}=u^{K},\ \mathfrak{s}\hat{u}^{K}=-(\partial A^{K}+m^{2}\phi^{K}) \tag{10}$$
$$\mathfrak{s}B:=i[Q,B],\ Q=\int d^{3}x(\partial^{\nu}A_{\nu}^{K}+m^{2}\phi^{K})\overleftrightarrow{\partial}_{0}u^{K}$$

Q is the so-called ghost charge (associated to a conserved ghost current) whose properties ensure the nilpotency ($\mathfrak{s}^{2}=0$) of the BRST $\mathfrak{s}$-operation. The A_{μ}^{K} is a point-like massive vector meson in the Feynman gauge and ϕ^{K} is a free scalar field whose Krein space two-point function has the opposite sign (a kind of negative metric scalar Stückelberg field). The "ghosts" $u,\hat{u}$ are free "scalar fermions" whose presence is necessary in order to recover the perturbative positivity of local observables and a unitary S-matrix.

The $\mathfrak{s}$-invariance of the scattering S-operator in gauge theory and the d_{e}-independence in the string-local setting are both related to BRST cohomology; but whereas the former has no relation to spacetime, the d_{e} acts on the space-like string directions of the in e independently fluctuating fields.

Since there are string-local fields with $d_{sd} = 1$ for all spins, there also exist string-local interaction densities within the power-counting limit $d_{sd}^{int} = 4$. But as already mentioned in the introduction, there is another physical restriction which has no counterpart in the point-local case: couplings of string-local fields are only physical if their higher order extensions preserve string-localization and if there remains a subalgebra of pointlike generated local observables.

This requirement, which plays no role within the point-local renormalization formalism, severely restricts interactions involving sl fields so that some of the advantage of low short distance dimension is lost. In the following this will be illustrated in three examples involving massive vector mesons (all fields are free fields). The first two models describe a massive vectormeson which couples either with a complex scalar field φ (scalar "massive QED") or with a Hermitian scalar field H

$$L^P = gA_\mu^P j^\mu, \quad j_\mu =: \varphi^* \overleftrightarrow{\partial}_\mu \varphi : \tag{11}$$

$$L^P = g : A_\mu^P A^{P,\mu} : H \tag{12}$$

In both cases A_μ^P is the point-local $d_{sd} = 2$ Proca potential so that the point-local interaction density L^P violates the PCB restriction of renormalizability.

Using the relation between the Proca potential and its string-local counterpart A_μ including its escort field ϕ (4), one may rewrite the nonrenormalizable point-local interactions into a $d_{sd}^{int} \leq 4$ string-local expression plus the divergence of another operator V_μ.

$$\begin{aligned} &L^P = L - \partial^\mu V_\mu, \quad L = gA_\mu j^\mu, \\ &V_\mu = j_\mu \phi, \quad j_\mu =: \varphi^* \overleftrightarrow{\partial}_\mu \varphi : \end{aligned} \tag{13}$$

The second line presents the wanted pair L, V_μ for massive scalar QED, for both operators $d_{sd} = 4$. The renormalization-preventing point-local interaction density $d_{sd}(L^P) = 5$ has been separated into two string-local contributions in such a way that the renormalizability spoiling $d_{sd} = 5$ contribution has been collected into the divergence of V_μ. In massive QFTs such divergence terms may be disposed of in the adiabatic limit so that the first order S-matrix of the power-counting violating L^P is the same as that of its better behaved string-local counterpart L. Although far from obvious, this idea of disposing renormalizability-violating terms at infinity can be generalized to higher orders. It is not limited to the S-matrix, but also leads to the construction of polynomial bounded correlation function of string-local quantum fields (private communication by Jens Mund). In other words the sl perturbation theory complies with the localization properties of string-local Wightman fields.

There is widespread belief that for perturbative renormalization theory one needs (either canonical or functional integral) Lagrangian quantization. But this is not correct; even for pl perturbation theory one only needs a scalar interaction density L^P in terms of free fields. These free fields need not be Euler-Lagrange fields; rather any free field obtained from covariantization of Wigner's pure quantum unitary representation theory of the Poincaré can be used. In fact Lagrangians for most higher spin fields are not known and sl fields are never Euler-Lagrange. The Stückelberg-Bogoliubov-Epstein-Glaser perturbation theory is based on the causal iteration of the first order scalar interaction density $\not{L}$ made from local Wick-products of free fields. There are no infinities, but the iteration leads to a growing number of new parameters (counterterm parameters) whose number only remains finite in case of the PCB $d_{sd}(L) \leq 4$.

The L, V_μ pair for the H coupling is less simple[10] since now also L depends on ϕ

$$L = gm(A \cdot A^P H + \phi \partial H - \frac{m_H^2}{2} \phi^2 H), \; V^\mu = gm(\phi A_\mu^P H + \frac{1}{2} \phi^2 \partial^\mu H) \tag{14}$$

[10] The perturbative calculations are simpler if one replaces only as many A^P by A as needed to obtain $d_{sd}(L) = 4$.

In this case the verification of the identity (13) requires the use of the free field equation for H with mass m_H (m =mass of vector meson).

A similar $L, \partial V$ pair exists for self-interacting massive vector mesons, e.g.

$$\begin{aligned} L^P &= \sum \varepsilon_{abc} F_a^{\mu\nu} A_{b,\mu}^P A_{c,\nu}^P = L - \partial V \\ L &= \sum \varepsilon_{abc} \left\{ F_a^{\mu\nu} A_{b,\mu} A_{c,\nu} + m^2 A_{a,\mu}^P A_b^\mu \phi^c \right\} \\ V_\mu &= \sum \varepsilon_{abc} F_a^{\mu\nu} \left\{ A_{b,\nu} + A_{b,\nu}^P \right\} \phi_c \end{aligned} \tag{15}$$

For the verification one again one has to use the field equation which in this case reads $\partial^\nu F_{\mu\nu} = m^2 A_\mu^P$.

For the extension to higher orders it is helpful to express the point-local nature in terms of the differential form calculus on de Sitter space

$$d_e(L - \partial V) = 0 \tag{16}$$

The existence of such pairs with L within $d_{sd}^{int} \leq 1$ turn out to be the prerequisite for the existence of local observables within a string-local setting. They also prevent the higher order spread of localization over all of spacetime. It should however be emphasized that the construction of L, V_μ pairs is a problem which can be pursued independent of L^P. Whether a collection of free fields permits a (maximally quadrilinear) coupling L which can be completed to a L, V_μ pair is a well-defined mathematical problem within the setting of differential forms on $d = 1 + 2$ de Sitter space. Whereas L must stay within PCB, the d_{sd} of V_μ may have contributions above $d_{sd} = 4$; as long as these contributions do not lead to higher order short distance contributions to L beyond $d_{sd} = 4$ the model remains sl renormalizable.

The exactness of the zero form $L - \partial V$ in (16) is a rather restrictive localization requirement. Such pairs within the power-counting bound for L turn out to be unique (if they exist) modulo additive changes of V_μ terms with vanishing divergence. Since in massive models the divergence ∂V disappears in the adiabatic on-shell limit, the first order contribution to the S-matrix are equal

$$S^{(1)\sim} \int L^P = \int L \tag{17}$$

String-local L, V_μ pairs are the starting point for the perturbative construction of the e-independent S-matrix and correlation functions of string-local fields. The problem how to maintain string-localization in higher order perturbations is closely related to the problem of preserving the e-independence of the S-matrix. This leads to a normalization condition on higher order time ordered products of $L - \partial V$ which will be commented on in section 4.

Note that the L, V formalism is not directly applicable to $m = 0$ since ϕ and V_μ have no massless limit. Behind this formal problem there is a radical conceptual change (breakdown of Wigner-Fock particle Hilbert space, infraparticles in QED, QCD confinement) whose spacetime implications have remained outside of our conceptual understanding of QFT. In fact these problems are outside the physical range of gauge theory; whereas short distance properties of unphysical gauge dependent fields are believed to share their asymptotic behavior with those of their sl physical counterparts (in particular the QCD asymptotic freedom) one does not expect that problems related to confinement can be accounted for in gauge theory. Here the long distance fluctuations of string directions are expected to become important. Following Wightman's reconstruction theorem [12] the massless QFT should be reconstructed from the massless limit of the massive correlation functions thus avoiding direct questions concerning the fate of the Wigner-Fock particle space.

3 Wilson loops, Haag duality and the Aharovov-Bohm effect

Consider the spacelike Wilson loop for a string-local vector potential. In the massive case one obtains from (4)

$$\oint A_\mu(x,e)dx^\mu = \oint (A_\mu(x) + \partial_\mu\phi(x,e))dx^\mu = \oint A_\mu(x)dx^\mu,\ m > 0 \tag{18}$$

whereas in the massless limit the separate contributions to the integrand diverge and instead one finds

$$\oint (A_\mu(x,e) - A_\mu(x,e'))dx^\mu = \oint \partial_\mu(\phi(x,e) - \phi(x,e'))dx^\mu = 0 \ \ for\ m = 0 \tag{19}$$

It is important to notice that, although neither ϕ nor $\partial_\mu\phi$ possess massless limits, the mass singularities cancel in the difference between $\phi's$ with different e-directions. This can be seen either in terms of the e-dependence of the intertwiners or by using the fact that the m^{-2} in (5) cancel in the 2-pointfunction of the difference

$$\langle\psi(x;e_1,e_1')\psi(x;e_2,e_2')\rangle\,, \quad \psi(x;e,e') := \phi(x,e) - \phi(x,e') \tag{20}$$

The e-independence of the loop integral despite its e-dependent integrand is reminiscent of its gauge invariance in the Krein space setting of point-local vector potentials. Later we will return to this analogy.

For the following it is convenient to work with operators instead of the singular quantum fields. A regularization of the vector potential in terms of a convolution with a smooth function f ,which is localized around a small ball B at the origin, leads to the regularized loop operator

$$\oint A_\mu^{reg}(x,e)dx^\mu, \quad A_\mu^{reg}(x,e) := \int f(x-x')A_\mu(x',e)d^4x' \tag{21}$$

It commutes with all operators whose localization region $\mathcal{O}$ is such that there exists a direction e for which the regularized half-cylinder does not intersect $\mathcal{O}$. This includes in particular all *convex* regions which do not intersect the torus l^{reg} which results from regularizing the loop l.

Operators whose localization region is such that there exists no choice of e which permits to avoid an intersection with the regularized semi-infinite cylinder $l^{reg} + \mathbb{R}_+ e$ do not commute with the regularized Wilson loop. This includes in particular operators which are localized in a torus which loops through l^{reg} without intersecting it.

By allowing the e to vary along the Wilson loop such that $e(\alpha)$ moves through a loop on the directional de Sitter spaces as $x(\alpha)$ sweeps through the Wilson loop, one enlarges the possibilities of avoiding intersections; but in case of a torus which intertwines l^{reg} without touching, an intersection with the e-extended Wilson loop is unavoidable. The dependence on e is "topological"; the Wilson "remembers" that its integrand had a directional dependence but it forgets in which direction it pointed.

This problem can be investigated directly in terms of the localization property of the electromagnetic field strength $F_{\mu\nu}$ without using vector potentials [16]. The result is that the operator representing a regularized magnetic flux through a surface D does not change under deformations of D as long as its boundary ∂D stays the same. Any operator which is localized in a contractible region outside the regularized torus $\partial D + B$ commutes with the flux operator but, as shown in [16], there are operators associated with interlocking but non intersecting toroidal regions which do not commute with the regularized magnetic flux operator. The authors refer to such a situation as the "breakdown of Haag duality".

Recall that Einstein causality states that two operators commute if their localization regions are spacelike separated. In terms of operator algebras this means

$$\mathcal{A}(\mathcal{O}) \ \subseteq\ \mathcal{A}(\mathcal{O}')', \quad Einstein\ causality \tag{22}$$
$$\mathcal{A}(\mathcal{O}) \ =\ \mathcal{A}(\mathcal{O}')', \quad Haag\ duality \tag{23}$$

where the dash on the region refers to the causal complement and that on the operator algebra to its commutant. Our intuitive understanding of causal localization is however in terms of Haag duality [13]; we expect that an operator which commutes with *all* algebras which are localized in the causal complement of a region $\mathcal{O}$ is really localized in $\mathcal{O}$ i.e. is a member of $\mathcal{A}(\mathcal{O})$.

Haag duality holds for all algebras which are generated by *massive* free fields and is believed to remain valid for observable subalgebras localized in multiply connected regions. But the properties of magnetic fluxes in QED show that Haag duality is broken for multiply connected subalgebras generated by point-local $s \geq 1$ massless field strengths $(F_{\mu\nu}, R_{\mu\nu\lambda\kappa}, ..)$. In other words there are operators in $\mathcal{A}(\mathcal{O}')'$ which do not belong to $\mathcal{A}(\mathcal{O})$ and the Wilson loop operator with its topological e-dependence is an example. Interestingly these breakdowns of Haag duality happen in theories in which the potentials are necessarily string-local.

In the gauge theoretic setting the point-local nature of vector-potentials "feigns" a localization on ∂D and in this way misses the breakdown of Haag duality. *The causal localization in gauge theories is not the physical localization.* What is missing for the case at hand is a property of the integrand of a Wilson loop which indicates that the regularized loop operator has the same Haag-duality breaking property as the magnetic flux operator.

The violation of Haag duality is basically a classical phenomenon. It is well-known that commutation properties of free quantum fields correspond to "symplectic orthogonality" of their corresponding wave functions

$$iIm(f,g) = [A(f), A(g)]\,,\ f,g\ real\ test\ functions \tag{24}$$

Hence the quantum A-B effect passes to its classical counterpart; the correct classic vector potential is simply the expectation value of its quantum counterpart in a suitable coherent state. The Stokes theorem does not contain informations about *physical* localization properties of vector potentials.

The "quirky" feeling that there may be some problems with causality in the A-B effect [11] has its origin in the naive identification of the gauge theoretic quantum causality in Krein space with that of a QFT in Hilbert space. The magnetic flux operator in a Hilbert space setting can be described in terms of covariant vector-potentials, but these are necessary string-local and even after the loop integration they retain a "topological memory".

The main reason for calling the readers attention to these facts (well-known among experts) is that the unphysical aspects of the quantum gauge formalism are not limited to problems of positivity (unitarity) but they also affect the foundational causal localization principles. The correct localized charge-carrying operators are obtained by smearing directional extended Wightman fields $\Psi(x,e)$ with compactly supported test functions in x and e and not those obtained by smearing gauge dependent pointlike fields.

The Hilbert space description of massless vector-potentials is traditionally presented in the form of Coulomb (or radiation) gauge. Being the unique Hilbert space potential which is rotation invariant in the $t=0$ hyperplane, it is not surprising that it is obtained from integrating the string-local potential over all string directions e in the hyperplane. The lack of covariance prevents its application in renormalized perturbation theory but does not impede its use in quantum mechanics.

The phenomenon of breakdown of Haag duality is a general property of all zero mass higher spin fields. For $s=2$ there are two string-local candidates which can be viewed as the analogs of the string-local A_μ namely the string-local $g_{\mu\nu}(x,e)$ (8) or the string-local $A_{\mu\nu\kappa}(x,e)$ which results from a line integration of the field strength $R_{\mu\nu\kappa\lambda}$ (9). The latter plays the analog role to that of the vector potential for $s=1$ in the verification of the breakdown of Haag duality. It is an interesting question whether in the Platonic world of duality violation there exists a relation between the multiple connectivity (the genus) of the spacetime localization region and the spin of the string-local zero mass potential. It also would be interesting to

[11] In most articles on the A-B effect the reader is assured that they are unfounded, but they certainly play a role in its popular appeal.

understand in what sense the construction of covariant string-local potentials can be viewed as a special case of recent constructions in [17].

The important role of positivity for infrared aspects of the QED Hilbert space can be seen by looking at simpler infrared problems in two-dimensional models. The simplest such model is the derivative coupling $\bar\psi\gamma_\mu\psi\partial^\mu\varphi$ of a $d=1+1$ fermion to the derivative of a $m>0$ scalar massive field φ [18]. It has a solution in terms of a formal exponential expression[12]

$$\psi(x)=e^{ig\varphi}\psi_0(x),\ \left\langle e^{ig\varphi(x)}e^{-ig\varphi(y)}\right\rangle=\exp g^2 i\Delta^+(x-y),\ \left\langle e^{ig\varphi(x)}e^{ig\varphi(y)}\right\rangle=\exp -g^2 i\Delta^+(x-y) \quad (25)$$

$$\exp g^2 i\Delta^+(\xi)=F(\xi^2m^2)\overset{m\to 0}{\to}(\xi^2m^2)^{-g^2},\quad \phi:=\lim_{m\to 0}m^{g^2}e^{ig\varphi},\quad \langle\phi\phi^*\rangle\neq 0,\ \langle\phi\phi\rangle=0 \quad (26)$$

Although φ itself has no positivity-preserving massless limit, the two-point correlation functions of its exponential field ϕ remain finite (after rescaling with a g-dependent mass factor) and and are consistent with conservation of the "ϕ-charge". In fact the combinatorial structure of the n-point function of ϕ in terms of exponential $i\Delta^+(x_i-x_k)$ contraction reveals that ϕ is a "g-charge" conserving field in the Hilbert space which the Wightman reconstruction theorem associates with the limiting ϕ vacuum expectation values [12].

There are two aspects of this construction which are worth mentioning. Whereas in the massive case the Hilbert space of the full model was a tensor product of Wigner-Fock φ-particle space with a ψ_0 particle space this structure gets lost in the massless limit since the g-charge creating ϕ field is not a member of the φ-Hilbert space. rather ϕ creates charge sectors in which the charge-neutral $\partial\varphi$ (the limiting φ does not exist) acts. Since the ϕ always appears together with the ψ_0, the Hilbert space is actually a subspace of the tensor product of the ψ_0 with the ϕ-space, i.e. the g-charge of ϕ coalesces wth the global ψ_0 charge. This in turn leads to a kind of kinematical infraparticle structure which manifests itself in the absence of the mass-shell delta function; the representation of the Poincaré group in the Hilbert space created by the application of $\psi's$ to the vacuum contains no discrete one particle state but instead of a mass-shell delta functions one finds a weaker threshold like singularity with a cut structure.

Suppose we ignore the $m\to 0$ limiting structure of exponentional and define a free φ_0 function logarithmic two-point function

$$\langle\varphi_0(x)\varphi_0(y)\rangle=\log\mu^2(x-y)^2$$

In this case the formally associated free field violates positivity i.e. the correlation functions define a linear indefinite metric space. In this case there is no charge superselection structure. The situation resembles that of the use of the Krein space point-local vector potential except that gauge theory restores perturbative positivity for certain (gauge-invariant) operators. But a structural understanding of problems behind infrared divergencies of pl charge-carrying operators within GT is not possible. In the SLFT setting *the singular pl siblings* $\psi(x)$ *disappear* in the massless limit and the Maxwell charge-carrying fields fields exist only as sl fields $\psi(x,e)$. At this point the "fake" matter fields $\psi^K(x)$ become fully misleading since the short distance compensation from negative metric contribution feigns a non existing pl localization; in this case the Hilbert space positivity does not even allow the existence of the singular pl siblings of the $\psi(x,e)$.

The understanding of long range properties of electric charges and the infraparticle aspect in QED pose more demanding dynamic challenges which go far beyond the kinematical observations on two-dimensional models. There remains however a formal analogy with properties one expects in the sl Hilbert space formulations. In order to highlight these analogies it is helpful to reformulate the previous observation by viewing the massless ϕ-fields as limits of spacelike line-integrals. With $j_\mu=\partial_\mu\varphi$ we may define the charge-carrying field ϕ directly in the massless model

[12] For reasons of brevity we omit the Wick-ordering in field products at the same point.

$$e^{-ig\varphi} = lim_{\Lambda\to\infty} e^{ig\int_0^\Lambda d\lambda j_\mu(x+\lambda e)e^\mu},\ e^2 = -1 \tag{27}$$

This suggests the following analogy ($\tilde{j}^\mu = \varepsilon^{\mu\nu} j_\nu$)

$$\exp ig\varphi(x,e) \sim \exp ig\Phi(x,e,e') := \Psi \tag{28}$$

$$with\ \varphi(x,e) = \int_0^\infty \tilde{j}_\mu(x+\lambda e)e^\mu d\lambda\ and\ \Phi(x,e,e') = \int_0^\infty A_\mu(x+\lambda e', e)e'^\mu d\lambda$$

In both cases the fields are logarithmically infrared divergent in the massless limit, whereas the exponential operators (defined as above by Wightman reconstruction from the massless limit of vacuum expectation values) remain finite.

In analogy to the ϕ charge we would like to view a state created by Ψ as carrying a Gauss charge i.e.

$$Q\left|\Psi\right\rangle := \int d^3x \vec{\nabla}\vec{E}\left|\Psi\right\rangle = \lim_{S\to\infty}\oint_S \vec{E}d\vec{S}\ \left|\Psi\right\rangle = \lim_{S\to\infty}\oint_S \left[\vec{E},\Psi\right] d\vec{S}\left|0\right\rangle \neq 0\ ? \tag{29}$$

Clearly such a superselected state cannot be compactly localized. Using exponential line integrals over point-local gauge potentials fails, since indefinite metric is not compatible with charges superselection rules, whereas the above ansatz has a better chance. In analogy to the 2-dim. model the full one-electron state should be of the form $\psi_0(x)\Psi(x.\infty)\left|0\right\rangle$ where ψ_0 is the free electron field and x in Ψ refers to the start of the semi-infinite strings.

One of the few rigorous results in QED is a theorem that the Lorentz symmetry is spontaneously broken in sectors of nontrivial electric charge [21]. This certainly does not happen in interactions with massive vector mesons. The heuristic picture is that the strings of charged particles are the centers of regions of noncompact infrared photon clouds. This is consistent with the established fact that such photon clouds leads to continuously many directional superselection sectors within a fixed charged sector [13]. Whether the above Ψ-states have this property can be checked by studying the energy-momentum density between such states.

Returning to the question of the structure of the Hilbert space one may summarize the present situation as follows. Independent of the pl or sl field localization the Hilbert space of asymptotically complete theories with a mass gap is a Wigner-Fock particle space where the particles are related to the interacting fields by LSZ scattering theory. This is a very clear conceptual situation. In massless limits involving sl $s \geq 1$ potentials this picture breaks down; in such cases the structure of the Hilbert space is expected to involve massless limits of nonpolynomial (in the case of QED exponential) string-local new sector creating composites of massless potentials. Fields which cannot be described as limits of free fields are outside the constructive range of QFT, and a Hilbert space description in terms of such fields could hardly be useful. But even in case the above proposal to describe the Hilbert space of QED is correct there remains the problem how these fields and their states are related to the large time limits of interacting fields in the sense of collision theory. Such constructions which aim at a better understanding of the momentum space recipes [22] [28] in terms of spacetime concepts of collision theory may be helpful.

A intriguing proposal can be found in a recent article by Buchholz and Roberts [23]. These authors observe that a restriction of the Minkowski space to a forward light cone V_+ would still permit a complete description of QFTs with a mass-gap, but its irreducibility would be lost in the presence of photons. In the context of string-local fields this situation suggests to use fields localized on spacelike hyperbolic curves which stay inside V_+ and only touch the surface of the light cone at lightlike infinity. Such a situation could lead to a more natural way to implement infrared cut-offs. What is missing is a perturbative realization of this idea; but the increasing perturbative experience with fields localized on spacelike or lightlike[13] lines suggests that such an extension may be possible.

4 Differential-geometric control of directional fluctuations

Whereas setting up first order string-local interactions in the form of L, V_μ pairs within the power-counting restriction is basically a kinematic problem involving free fields, the situation changes when it comes to the construction of the higher order S-matrix. The reason is that the singular nature of time ordering does not permit to take the divergence $\partial^\mu T(..\partial_\mu\varphi..)$ directly through the time-ordering to the affected operator. The differential relation[14]

$$\begin{aligned} &d(TLL' - \partial^\mu TV_\mu L' - \partial'^\mu TLV'_\mu + \partial^\mu\partial'^\mu TV_\mu V'_{\mu'}) = 0, \quad d = d_e + d_{e'} \\ &TLL'|^P := TLL' - \partial^\mu TV_\mu L' - \partial'^\mu TLV'_\mu + \partial^\mu\partial'^\mu TV_\mu V'_{\mu'}, \end{aligned} \tag{30}$$

which secures the e-independence of the second order S-matrix in the adiabatic limit (formally the integral over Minkowski spacetime)[15], would be a trivial consequence of (16) if it were not for singularities in T-products from coalescent points and crossing of strings.

In terms of differential forms in the de Sitter space of directions the individual contributions to the right hand side are zero forms and their sum is an exact. We remind the reader that in the presence of a mass gap there are no boundary terms at infinity, so that in analogy to (17) the divergence terms do not contribute to the second order S-matrix

$$S^{(2)} \sim \int\int TLL'|^P = \int\int TLL' \tag{31}$$

It turns out that such "normalization" problems as posed by (30) can be solved by using the freedom in defining time-ordered products. Starting from a "kinematic" time ordering T_0, one computes the anomaly A as the singular part of the terms containing derivatives

$$-A = s.p.(-\partial^\mu T_0 V_\mu L' - \partial'^\mu T_0 L V'_\mu + \partial^\mu\partial'^\nu T_0 V_\mu V'_\nu) \tag{32}$$

Since in the following we will be interested in the S-matrix, we only need the contribution from the 1-contraction (the tree component) $A|_{1-contr.}$; for notational economy we will omit the subscript, so in the following relations the A stands for the tree component of the anomaly.

The kinematic T_0 is defined by taking all derivatives outside e.g.

$$\begin{aligned} \langle T_0\partial\varphi\partial'\varphi^{*\prime}\rangle \quad &: \quad = \partial\partial'\langle T_0\varphi\varphi^{*\prime}\rangle \\ \partial^\mu\langle T_0\partial_\mu\varphi\varphi^{*\prime}\rangle \quad &= \quad -i\delta(x-x') - m^2\langle T_0\varphi\varphi^{*\prime}\rangle \end{aligned} \tag{33}$$

If the anomaly is of the form

$$A = -N + \partial^\mu N_\mu + \partial^\mu\partial^\nu N_{\mu\nu} \tag{34}$$

where the $N's$ contain $\delta(x-x')$ functions, they can be absorbed as renormalization terms in the time-ordered operator products (30). They describes the delta function terms which violate the relation (30) if

[13] I am indebted to Jens Mund for informing me that the use of lightlike instead of spacelike linear strings causes no new problems.

[14] Unpublished remark by Jens Mund (Vienna 2011).

[15] In massive theories boundary terms at infinity vanish. Without x,x' integration the expression in the bracket can be used as a definition of a second order point-local interaction density..

one uses the kinematical T_0; hence the anomaly terms reveal in what way the time ordered products at coalescent points (or more general at string intersections) in (30) have to be defined.

For the calculation of the second order S-matrix one only needs to compute the N. Note that N terms are similar to the counterterms well known from the renormalization formalism for point-local interactions. But there is a significant conceptual difference. Whereas the renormalization counterterms in point-local renormalization come with new coupling parameters, the contact terms originating from anomalies are uniquely determined in terms of the basic first order couplings and the masses of the free fields in terms of which the first order interaction density is defined. Such anomaly terms will be referred to as *induced interactions*. They originate *from the implementation of the e-independence of the S-matrix* and hence they have no analog in $s<1$ point-local interactions.

The calculation of the S-matrix requires only the calculation of N. In that case it is more convenient to use a weaker formulation

$$\begin{aligned} &dL-\partial^\mu Q_\mu=0,\ Q_\mu:=d_eV_\mu \\ &dTLL'-\partial^\mu TQ_\mu L'-\partial'^\mu TLQ'_\mu=0 \end{aligned} \tag{35}$$

As will be seen later, this "Q-formulation" is closely related to the implementation of the gauge invariant S-matrix in the $sS=0$ in the CGI BRST setting.

The V-formalism, which leads to the definition of pointlocal interactions densities (30), turns out to be indispensable for the construction of interacting string-local fields. It permits to define higher order point-local interaction densities in terms of the renormalizable string-local formalism. Together with the formal relation between point- and string-local matter fields (6) it can be used for the perturbative construction of renormalized correlation functions of string-local fields. The point-local higher order interaction densities (30) play the role of the e-independence of string-local field correlations from the $e's$ of internal propagators. This construction of interacting string-local fields has been initiated by Jens Mund and will be the subject of a forthcoming publication.

Assuming that these string-local fields remain renormalizable (Wightman fields in x) in every order, the results of Jaffe [15] on the singular nature of exponentials of scalar free fields suggest that $\psi^P(x)=\psi(x,e)e^{-ig\phi}$ is a singular (not Wightman-localizable) field in a well-defined theory. As previously mentioned it shares its bad short distance properties (unbounded increase of d_{sd} with the perturbative order) with that of the field in the point-local setting, but at least its singular behavior is not accompanied by a (with perturbative order) growing number of coupling parameters. The bad high enery behavior of the off-shell vacuum expectation of the nonrenormalizable pl Hilbert space setting remains, but the numerical coefficients of the worsening pl counterterms do not introduce new parameters.

In this situation one expects that there are high energy on-shell cancellations which permit the scattering amplitudes of the singular pl fiels to be the same as those obtained from the renormalizable sl correlations. In such models the cause of pl nonrenormalizability is a weakening of localization. Assuming that these observations are correct, the lack of renormalizability either indicates a weakening of localization or the PCB violating coupling does not define a QFT.

Another important new concept which has no analog in $s<1$ pl interactions is the before-mentioned appearance of *induced terms*. In the following this will be illustrated in terms of sl second order calculations three different models.

The first such model is scalar massive QED (13). In the case the anomaly contribution arises from the divergence acting on the two point function involving a $\partial_\mu\varphi$. The result for the induced contact contribution is the expected second order quadratic in A_μ contribution

$$g^2:\varphi^*(x)A_\mu(x,e)\delta(x-x')\varphi(x')A^\mu(x,e'):+(e\leftrightarrow e') \tag{36}$$

which may be absorbed into a change of $T_0 \rightarrow T\!\!\!/$ product of the $\partial\varphi\partial'\varphi^{*\prime}$ contraction contributing

$$\langle T_0 \partial_\mu \varphi^* \partial_\nu \varphi \rangle \rightarrow \langle T \partial_\mu \varphi^* \partial_\nu \varphi \rangle = \langle T_0 \partial_\mu \varphi^* \partial_\nu \varphi \rangle + c g_{\mu\nu} \delta(x - x')$$

For more details we refer to [8].

This is similar to gauge theory where it results from BRST gauge invariance of the S-matrix[16] $\mathfrak{s}S = 0$, except that the independence of the S-matrix from string directions is a natural physical consequence of large time scattering theory [6] for string-local fields in the presence of a mass-gap. In a perturbative setting, which is based on the construction of the S matrix in terms of the adiabatic limit of Bogoliubov's formal time-ordered products of interaction densities, it has to be imposed (30).

There is a fine point which turns out to be of significant conceptual importance. The directional fluctuations only disappear after adding up all contributions to a particular scattering amplitude to the same perturbative order. For the case at hand the singularity at $e = e'$ in the time-ordered propagator (which enters the second order tree contribution to scattering) corresponding to the two-point function (5)

$$\frac{1}{p^2 - m^2 - i\varepsilon}(-g_{\mu\mu'} - \frac{p_\mu p_{\mu'}(e \cdot e')}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)} + \frac{p_\mu e_{\mu'}}{(p \cdot e - i\varepsilon)} + \frac{p_\mu e'_{\mu'}}{(p \cdot e' + i\varepsilon)})$$

is ill-defined since the two fluctuating directions enter with a different ε-prescription[17]. But in the use of this propagator for the second order *on-shell* scattering amplitude (the scalar analog of Møller- and Bhaba-scattering) this problem disappears. This is similar to the verification of on-shell gauge invariance in the second order tree approximation except that e, e' are not global gauge parameters in an Krein space gauge theory but rather spacelike directions of independently fluctuating string-local vector potentials acting in a Hilbert space.

Whereas in the Krein space gauge setting individual perturbative contribution to a scattering process exists regardless whether it is gauge invariant by itself or not, the independently fluctuating $e's$ become only "mute" after adding up sufficiently many perturbative on-shell contributions in a fixed order. Any attempt to interpret the independently fluctuating $e's$ as gauge parameters by equating them in contributions from different fields causes the renormalization-resistent infinities of the abandoned axial gauge formalism to return with full vengeance.

It is important to notice that the second order $A \cdot A\varphi^*\varphi$ contribution is induced by the principles of QFT (causal localization in Hilbert space), there is no reference to a classical gauge structure. This is even more remarkable in models of self-interacting vector mesons in which the Lie-algebra structure, unlike classical gauge theory, bears no relation to an imposed symmetry but rather results from only implementing the spacetime causality principles. That the classical causality principles of Faraday, Maxwell and Einstein become such powerful in the context of Hilbert space positivity of interacting higher spin $s \geq 1$ quantum matter is truly surprising.

A much richer second order induction occurs in the $gA \cdot AH$ coupling of massive vector mesons to a Hermitian matter field H (14). In this case the requirement of second order e-independence (30) of S for the more involved first order $L, \partial V$ leads to a larger number (no charge symmetry) of induced interactions which includes the H^4 self-interaction and even requires the presence of a first order modification by H^3 self-interactions. The sum of both contributions has the form of a Mexican hat [8] just as in the CGI setting in [9], where the numerical coefficients contain besides the coupling strength g also mass ratios of the two physical masses m_H, m.

[16] In the naive formulation the quadratic term arises from the replacement of $\partial \rightarrow \partial - igA$ in order to preserve gauge invariance of the classical Lagrangian.

[17] This is the reason why the axial gauge interpretation failed.

These *induced* "Mexican hat" terms would also appear in the BRST gauge formalism but, as already mentioned before, in a diagrammatic Feynman formalism it is difficult to be aware of the differences between on- and off-shell properties. Implementing the on-shell BRST $\mathfrak{s}$-invariance $\mathfrak{s}S = 0$, or the e-independence $dS = 0$ in the SLF Hilbert space setting, it is impossible to overlook the *induction* of a Mexican hat like self-interaction or confuse it with an off-shell SSB mechanism; it is the massive vector meson which is in the driver's seat and not the scalar matter coupled to it [9]. Apart from the richer structure of the induced Mexican hat potentials (related to the absence of the charge selection rule for H-couplings), this is analog to the second order induction of the quadratic A-dependence[18] in massive scalar QED. In both cases the only conserved current is the "massive Maxwell current" $j_\mu^{Max} = \partial^\nu F_{\mu\nu}$ and its charge is always screened ($Q^{Max} = 0$) and not spontaneously broken ($Q^{SSB} = \infty$) [8] (and references therein).

It is interesting to note that unitarity properties of tree graphs and a bit of common sense, ignoring classical gauge structures, led some authors [24] [25] to the correct result already in the early 70's; including the idea that the Lie-algebraic structure of self-interacting vector-mesons may be a natural property of self-interacting massive vectormesons and needs no imposition of gauge symmetry. If they had formulated their unitarity requirements directly in terms of interactions of massive vector mesons with Hermitian matter fields they would have noticed that there is no need to invoke a SSB Higgs mechanism. Pushing a bit harder they could have seen that the reason why massive QCD or just Y-M (as opposed to massive QED) requires the presence of a H coupling just in order to avoid violations of the second order PCB (and not for implementing SSB). The present work on interactions of $s = 1$ vector mesons in a Hilbert space setting can be seen as a field theoretic confirmation of these on-shell unitarity arguments which need neither the invocation of gauge symmetries nor that of SSB.

It is certainly true that the formalistic "breaking of gauge invariance" by a shift in field space of scalar QED combined with an hermiticity-restoring gauge transformation leads to the abelian Higgs model (what else could it do?). *But this bears no relation to the intrinsic properties of the resulting model.*

In any renormalizable model the form of the local interaction density is uniquely determined in terms of the field content (including the physical masses of the free fields) and the imposed internal symmetries. Renormalization theory leads to additional contributions in the form of counter-terms with new coupling parameters. The new insight is that the preservation of Hilbert space positivity in the presence of $s = 1$ vector-potentials leads to the phenomenon of *induced* interactions; in contrast to the free parameters of counter-terms (which are only subject to the imposed inner symmetries), *induced contributions do not enlarge the number of free parameters.* In particular the numerical coefficients of the induced Mexican hat potential are determined in terms of the $A \cdot AH$ coupling strength g and the two masses m, m_H.

SSB in the sense of Goldstone ($\partial^\mu j_\mu = 0$ but $Q = \infty$) requires an *interaction density which contains massive as well as massless scalar free fields in a precisely tuned proportion.* A helpful way to obtain such a situation is to start from the conserved current of a symmetrically coupled self-interacting scalar multiplet with a symmetric mass term of the opposite sign (so the potential takes the form of a Mexican hat). It is well known that a suitable shift in field space brings the minimum back to the value zero. One then checks that this shift manipulations changes the form of the field equation but the conservation of the current (which uses the field equation) remains intact. The new field equations contain mass terms which depend on the choice of the direction of the field shift in the multi-component field space, but there always will be some zero mass particles (the Goldstone bosons).

After the semiclassical preparation of the model, QFT (in the form of renormalized perturbation theory) takes over. It maintains the physical masses of the interaction density defining free fields and the conservation of the current in every order of perturbation theory. Some of the integrals defining the

[18] There is no reason to refer to the $\partial \to \partial - ieA$ classical gauge connection, the quadratic A term results solely from the quantum principles of QFT..

conserved charges (the would be global symmetry generators) are prevented from converging by long range manifestation of the Goldstone bosons. This (and not the quasiclassical manipulations to obtain such a QFT) is the *definition of SSB* (the non-invertibility of the Noether theorem i.e. $\partial j = 0$ but $Q = \int j_0 = \infty$). An alternative more general picture is to say that there are "partial" charges localized in a causally closed spacetime regions whose exponentiated unitary symmetry operators implement the full symmetries on observables in a smaller spacetime region. Some of these partial charges fail to be extendible to charges which implement global unitary symmetries. For a model-independent structural theorem about SSB see [26].

QFT is not a theory which says anything about the masses of elementary free fields which enter the definition of a model. Masses of bound states interpolated by composite fields on the other hand are expected to be computable (although there is no known consistent perturbative method to implement this. The terminology "SSB creation of masses" is still within the range of tolerable metaphors of particle physics. The danger of misunderstandings starts if these manipulations are applied outside their range of validity.

As explained previously for interactions involving higher spin fields there are restrictions (gauge invariance, directional independence of S) which lead to the induction of H selfinteraction. Starting from $A \cdot AH$ interactions between a massive vector meson and a Hermitian field one obtains the same Higgs models as from the SSB Higgs mechanism which starts from a massless gauge theory of a complex field. Which is the correct derivation i.e. the derivation which is consistent with the physical principles? The answer is that the one based on the gauge invariance of the S-matrix or even better on e-independence of S is consistent with QFT principles. The reason is obvious since SSB, even there where it is meaningful is an option whereas gauge invariance or the properties of SLFT are part of the conceptual structure of interacting massive vector mesons.

QFT is a foundational quantum theory which reveals its intrinsic properties independent of by what computational tricks or thoughts the acting physicist obtained a model. In case of SSB the integral over the charge density of the conserved current diverges (the definition of SSB) and in models involving massive vector mesons the charge is "screened" i.e. the Q vanishes. This is obviously a generic property of an identically conserved Maxwell current $j_\mu = \partial^\nu F_{\mu\nu}$ [8]. One should not allow to confuse such opposite situations by metaphors as: "photons fattened by eating Goldstones".

One obtains a better understanding of the history of the Higgs model when one recalls that QED resulted from a natural adaptation of the classical Maxwell's theory to the requirements of charge-carrying (complex) quantum matter. That classical theory contains however no suggestion of how to couple vector potentials to *Hermitian* matter. As we know nowadays such couplings vanish in the limit $m \to 0$ i.e. they only exist in the presence of massive vector mesons. Therefore the SSB prescription of a shift in field space was a helpful formal device to become aware of a new coupling to Hermitian matter which disappears in the Maxwell limit. The problem started when this prescription was misinterpreted as a physical SSB of gauge symmetry which "creates" masses. This does not only reveal a misunderstanding of the role of the quantum gauge formalism (a method to recover physical properties from an unphysical Krein space setting), but also of SSB (a conserved current with a *diverging* charge).

However in *massive QCD* or just massive *Y-M* one *really needs a coupling to a H-field,* so the important remaining problem (the most important of the entire section) is to understand why for self-interacting massive vectormesons the consistency of the model requires the presence of $H's$, whereas in massive (spinor or scalar) QED they play no role.

GT and the Hilbert space based SLFT lead to the same answer to this question, though the details of the second order calculations are somewhat different. The gauge theoretic requirement $\mathfrak{s}S = 0$ [9], as well as the $dS = 0$ formalism derived from first principles of QFT induce nonrenormalizable second order self-interactions of $d_{sd} = 5$. *This is the only known case in which a first order interaction within the* *power-counting restriction $d_{sd}^{int} \leq 4$ turns nonrenormalizable in second order!* When in the old days the 4-Fermion model of weak interactions was replaced by the short-distance improved intermediate massive vector meson model in the setting of massive nonabelian gauge theory, it was at first overlooked that this model fails to preserve second order renormalizability. Hence the massive vector meson exchange converted Fermi's 4-Fermi coupling first-order PCB whereas the second order compensation with A-H interactions maintains this in second order.

The compensatory mechanism is reminiscent of compensations of short distance singularities between contributions from different spins in supermultiplets. The compensating field coupled to the self-interacting massive vector mesons should have a lower spin (a higher spin would make sense worse) and the same Hermiticity properties as the A_μ, hence it must be a H-field.

It turns out that the only way to preserve the second order power-counting restriction is to *enlarge the first order A self-interaction by a nonabelian $A \cdot AH$ coupling*[19]. Its second order contribution contains in addition to those term expected from the analogy with the abelian model an additional $d_{sd} = 5$ contribution which can be used to compensate the incriminated nonrenormalizable term from the self-interaction. The net result is that, apart from the induced Mexican hat potential, all non-compensating anomaly contributions can be absorbed into modified time-ordered products (propagators) $T_0 \rightarrow T$. There is no physical role for the H field to play in the understanding of massive abelian gauge theories, but their compensatory role in the presence of *self-interacting* massive vectormesons is indispensable.

This preservation of renormalizability by short distance compensations is reminiscent of cancellations between different spins contributions in supersymmetric interactions. The difficulty that one does not know how to formulate a supersymmetry-preserving renormalization theory is absent in case of gauge invariance or e-independence. The formal manipulations within a "Higgs mechanism" give the complete interaction in one swoop but fail to realize that behind the scene there is a new type of compensation between different spin contributions which different from those of supersymmetry. For the GT presentation of this compensation the reader should consult [9], some remarks about its SLFT derivation can be found in [8] and the details will be contained in joint work with Jens Mund.

The reason why it could be important to understand the correct physical cause for the presence of the H (apart from the fact that in a foundational theory as QFT it is always good to know the intrinsic reasons for a new phenomenon) is that the construction of L, V_μ pairs may lead to compensatory situations involving higher spin fields as $g_{\mu\nu}(x, e)$ together with compensatory $s = 1$ and $s = 0$ contributions. It would not be necessary that V_μ has $d_{sd} \leq 4$ as long as the higher short distance dimensions compensate in higher orders.

In the Higgs paper [27] one finds the remark that the H-field is needed to generate a mass. In this context there are references to works in condensed matter physics as the phenomenological Landau-Ginsberg description of superconductivity and its microscopic BCS refinement (also Anderson's work on energy gaps from a gap-less situation). In all those works already *existing degrees of freedom regroup themselves* (the Cooper pairs) and in this way lead to new physical manifestations. But this is precisely the mechanism by which the Hilbert space description of *massive* QED requires the introduction of a scalar string-local field ϕ (the $A'_\mu s$ "escort") which depends on the same degrees of freedom (namely the $s = 1$ Wigner creation/annihilation operators) as the massive vector potential.

As the BCS superconductivity cannot be described without the regroupment of condensed matter degrees of freedom in the form of Cooper pairs, massive QED in a Hilbert space setting needs the presence of the (degrees of freedom maintaining) escort field ϕ. Gauge theory in Krein space is a physical incomplete theory (no physical substitute for gauge-dependent fields) which has many additional unphysical degrees of freedom (negative metric fields, ghost fields) but no physical ϕ-field. The degree of freedom issue and

[19] The short distance compensation does nit fix the number of $H's$. In case of one H (which seems to be favored by LHC) the coupling is unique.

the appearance of additional fields is a quantum phenomenon which has no classical counterpart. Whereas the perturbative structure of $s < 1$ QFT maintains formal relations to classical FT, these links are lost in the presence of $s \geq 1$ fields.

Precisely for this reason it is important to understand the physical cause *why additional H-degrees of freedom are needed in the presence of self-interacting massive vector mesons.* It turns out that the first order modification of the 4-Fermi interaction by intermediate massive vector mesons leading a nonabelian massive gauge theory only secured the first order PCB but that the second order leads to a renormalizability-violating $d_{sd} = 5$ term. Since in all previous studied models of QFT the first order PCB bound $d_{sd} \leq 4$ always implied renormalizability[20], this phenomenon was overlooked. The interpretation of the nonabelian Higgs model as resulting from a SSB breaking of gauge invariance obscured this problem since the *physical* understanding of a second order short distance compensation mechanism cannot be delegated to formal description of a model as arising from SSB of gauge symmetry (a physical symmetry ?). In the more physical SLFT Hilbert space setting for $s = 1$ interactions there is no gauge symmetry, so what is the S to which the SSB prescription should be applied ?

One rather has to realize that the coupling of H fields to massive vector mesons defines autonomous theories and that the H-selfineractions (the Mexican hat potential) is "induced" and not the result of an imposed SSB. The induction phenomenon, which was explained in the beginning of this section, does not exist for point-local $s < 1$ interactions but is characteristic for interactions involving string-local $s \geq 1$ fields. Another phenomenon which appears only for $s \geq 1$ is that a massive model within first order PCB may violate second order PCB. In such a case one has to look for an extension of the model by adding an interaction with an additional field which leads to a second order compensation of the renormalizability threatening second order terms. Precisely this happens for self-interacting massive vector mesons in which the compensating fields are H fields; this, and not the alleged role of a mass creating midwife, is the foundational purpose of the H.

As far as the classification of all second order compensation of self-interacting massive vector mesons of different masses couples with H-multiplets with different mass is concerned the recipe based on SSB breaking schemes turns out to be quite efficient because the SSB formalism leads to the same family of models as the classification of compensations [11]. Nature does not follow the logic of ostensible simplicity of calculational prescriptions of particle theorists but rather sticks to her own principles. The relation of pertubative renormalizability with causal localization in a Hilbert space setting is certainly one of her deep but still insufficiently understood conceptual messages.

That important discoveries have been made by less than correct reasons is not new; even Dirac's argument in favor of anti-particles was based on the inconsistent hole theory. For the experimental discovery of the Higgs particle this made no difference; one may even speculate that the somewhat mysterious appeal of the SSB explanation (creating the masses of all particles, including its own) may have been a more important motivating force for the enormous mental and material effort which went into its discovery at LHC than the preservation of second order renormalizability (which to many particle physicists may appear as a dèjá vu of the 4-Fermi past).

What is a bit worrisome (as compared to the history of antiparticles and other important discoveries originally made for less than correct reasons) is that the SSB Higgs mechanism still dominates the scene after 4 decades. This raises the question whether explanations which found acceptance within a globalized community (apart from the tiny group of scholars who noticed conceptual inconsistencies) can be still be corrected in times of "Big Science". In any case the main purpose of the present work is not to address sociological problems but rather the presentation of a surprising new look at one of QFT's oldest problems: how to formulate $s \geq 1$ interactions in a quantum theoretical Hilbert space setting.

[20]Higher order short distance compensation between fields of different spin are only expected in SUSY QFTs.

This includes in particular problems involving massless vector potentials. The perturbative infrared divergencies for scattering of electric charge-carrying particles signal the breakdown of the spacetime understanding of the field-particle relation in terms of (LSZ, Haag-Ruelle) scattering theory. Successful momentum space recipes (the Bloch-Nordsieck prescription and its Yennie-Frautschi-Suura refinement [28]) for photon-inclusive cross sections cannot hide the fact that GT, as a result of absence of charge-carrying matter fields, is not able to solve this problem.

One expects that the new SLFT theory sheds new light on this age-old problem. After all passing form point-local fields in GT to covariant string-local interacting fields is not the result of a playful attitude to try out something else (as in ST), but rather the inevitable consequence of upholding one of quantum theory's most cherished principles: the positivity of the Hilbert space.

The correct analog of quantum mechanical long-range Coulomb potentials are string-local charge-carrying matter fields. The associated particles are still registered in compact localized counters; the only influence of noncompact field localization on particle counting events are spread over a larger spacetime region. As previously mentioned there exist rigorous results based on the quantum Gauss law which show the noncompactness of charge localization and the "stiffness" of string-local charge-carrying states whose directions (different from that of their massive counterparts) cause a spontaneous breaking of Lorentz-invariance [20] [21] and leads to a radical structural change of the Fock space (see end of previous section).

The only objects which remain relatively unaffected by the structural changes in the massless limit are the expectation values of string-local fields. Their calculation in the massive case requires the use of the point-local interaction densities $(TL..L)^P$ (30) as well as the relation (6) between point- and string-local matter fields. The extension of the on-shell S-matrix formalism to off-shell string-local fields is an ambitious program which has been initiated by Jens Mund. One expects that its leads to both renormalizable (Wightman-localizable) string-local and singular (Jaffe class) point-local matter fields. Only the string-local correlations are expected to possess a massless limit; they account for the properties of the "rigid" (SSB L-invariance-breaking) electric charge-carrying strings.

The QED Hilbert space and the string-local operators acting in it are then determined in terms of Wightman's reconstruction theorem [12]. They should provide the spacetime explanation for the change of the electron mass-shell into a milder cut-like singularity and the momentum space recipes which replace the vanishing scattering amplitudes of charge-carrying particles by the recipe for photon-inclusive cross sections.

A more radical behavior is expected in the massless limit of self-interacting vector mesons. The conjectured confinement means that only correlations which contain solely point-local hadron and gluonium composites as well as "string-bridged" $q-\bar{q}$ pairs will survive this limit whereas correlation containing string-local gluon or quark fields vanish. From the analogy with the vanishing QED scattering amplitudes for electrically charged particles with a finite number of participating photons one expects that there will be logarithmic infrared divergencies in the presence of gluon or quark operators whose summation of leading perturbative contributions for $m << 1$ will lead to functions which vanish in the massless limit.

But does this conjecture have a chance to be confirmed by perturbative calculations? Certainly not in GT, since off-shell correlation functions in covariant gauges are known to remain infrared finite [29]. But the directional fluctuations of string-local fields have stronger infrared ramifications than the point-local fields in covariant gauges of GT. They are expected to be especially strong in the presence of self-interacting vector potentials. The lowest order candidate for testing the possible presence of an off-shell logarithmic divergence for $m \to 0$ would be the e-dependent second order F-F propagator. The off-shell perturbation theory for string-local fields is presently under construction.

5 Gauge theory and local quantum physics

Despite significant conceptual differences between GT and SLFT there are also formal analogies. A comparison between these two ways of describing QFT of $s = 1$ fields leads to a better understanding of their mutual relation. In the absence of interactions the differences only affects a fine point in the interpretation of causality in gauge invariant Wilson loops (section 3), but they increase in the presence of interactions. The construction of the globally gauge invariant S-matrix is a good illustration of the conceptual differences between the two settings.

The CGI BRST operator formalism handles this problem in the following way. Instead of (4) one writes

$$A_\mu^K(x) - \partial_\mu \phi^K =: A_\mu^{P,K}(x) \tag{37}$$

where K refers to Krein space, A_μ^K is the massive vector-potential in the Feynman gauge, ϕ^K is a massive free scalar field with the opposite sign in its two-point function (the auxiliary "Stückelberg field") and $A_\mu^{P,K}$ is a substitute for the Proca potential. Whereas ϕ^K adds unphysical degrees of freedom, the SLFT sl escort ϕ results from a rearrangement of physical degrees of freedom[21] and disappear in the massless limit.

The u^K and $\hat{u}^K$ in (10) are added "ghost" degree of freedom which serve to formalize to incorporate the unitarity arguments of 't Hooft and Veltman into an operational formalism but, together with ϕ^K the remain without physical interpretation.

As noticed by Mund (private communication), the would-be Krein space Proca field has indeed the same two-point function as its Hilbert space counterpart; but being an object in a larger Krein space (the tensor product of two Krein spaces for $s = 1$ with $s = 0$ extended by the ghosts) it "leaks" into the tensor-product Krein space, i.e. its "one-particle vector" $|p\rangle^K$ is not a Wigner particle but represents the "best Krein space approximation" to one.

Using the ghost rules (10) one defines a L^K, Q_μ^K pair which fulfills [9]

$$\begin{aligned} &\mathfrak{s}L^K - \partial^\mu Q_\mu^K = 0, \ \textit{hence}\ \mathfrak{s}S^{(1)} = 0 \\ &\mathfrak{s}TL^K L^{K\prime} - \partial^\mu T Q_\mu^K L'^K - \partial'^\mu T L^K Q'^K_\mu = 0, \ \textit{hence}\ \mathfrak{s}S^{(2)} = 0 \end{aligned} \tag{38}$$

In analogy to the e-independence (35), but with the significant conceptual difference that, whereas *d acts on the individual spacetime string directions*, the globally acting BRST nilpotent $\mathfrak{s}$ has has no physical interpretation of its own, but serves to extract a physical S-matrix from an unphysical point-local Krein space description.

The formal analogy of the SLFT Q formalism (35) with (38) stands in contrast to the quite different behavior of the two $Q's$ in the massless. The gauge dependent zero mass matter (e.g. the electron field) is fictitious, there is no interacting physical pl electron field. The singular pl matter field which still existed in the massive physical sl setting disappears in the massless limit. Whereas the gauge formalism provides no warning concerning a fundamental conceptual change (the breakdown of the Wigner-Fock particle setting and scattering theory) the sl setting clearly signals this through the breakdown of the Q-formalism and the message that the only object which may survive the infrared limit are the correlation functions. This is a replay of the message that the only physical (positivy preserving) zero mass free field objects are the massless limits of the free sl massive correlation functions.

Since there is no presently known way to formulate collision theory in the presence of infrared problems, the only option is to use the Bloch-Nordsiek presriptions in the adaptation to QED in [28] which lead to

[21] Remember the analogy to Cooper pairs from rearrangements of condensed matter degrees of freedom which causes the short range nature of vector potentials in the superconducting phase.

photon-inclusive cross sections. A spacetime understanding of "infra-particles" and their collisions in QED within the principles of QFT (i.e. in terms of sl fields in Hilbert space) is still missing.

Whereas the application of the SLFT Hilbert space formalism is still in its initial stages, the CGI adaptation of BRST gauge theory already exists since the early 90s [9] (see [11] for a recent account). Its main purpose has been a clarification of the Higgs issue. That formulation of the BRST GT is formally close to the present Hilbert space setting. An attempt in [30] to use it as a platform for Hilbert space formulation of Wigner particles turned out to be premature; the central role of the clash between point-like localization and Hilbert space positivity and its resolution in terms of string-local fields was only recognized afterwards in [4] [31] [7] and more recently in [8].

The formal similarities between CGI and SLFT end when it comes to the construction of the e-independent interaction densities $(TL..L)^P$ in terms of their string-local counterparts (30). Together with (6) they enter in the construction of off-shell vacuum expectation values of string-local fields and their zero mass limits. This is still virgin territory within the reach of SLFT but outside the physical range of GT.

There are some puzzling observation which came from the CGI perturbative implementation of on-shell gauge invariance $\mathfrak{s}S = 0$. Quite unexpectedly one finds that if one start with the most general ansatz of an interaction density of mutually self-interacting massive vector mesons of the same mass (omitting the superscript K)

$$L = \sum f_{abc} F^{a.\mu\nu} A_\mu^b A_\nu^c \tag{39}$$

with arbitrary real coupling coefficient f_{abc}, the implementation of operator gauge invariance (38) up to second order requires these coefficients to satisfy the Lie-algebra relation of the adjoint representation. This continues to hold if one allows the masses to be different and uses the most general first order couplings within the power-counting restriction, including the H-coupling which one needs for the preservation of second order renormalizability. One obtains the same relations between couplings and masses as those obtained by imposing the formal SSB prescription on the massless gauge invariant Lagrangian [11], but this time from merely imposing the BRST gauge invariance (38) on the tree approximation up to third order.

At first sight I found this very impressive, but in phone conservations and email exchanges Raymond Stora (who sadly passed away in June this year) convinced me that one should not be surprised but rather view this as the imprint of classical gauge symmetry (connections,fibre bundles) on the BRST formalism. The real challenge, according to Stora, is the derivation of this Lie algebra structure from first principles of QFT.

In the new SLFT setting which is based on only first principles it is easy to formally derive the Lie-algebraic structure in the zero mass limit [8]. But since the Hilbert space $Q's$ diverge in the massless limit the result is at best a consistency argument. Only a derivation in the presence of self-interacting massive vector mesons can settle this problem (the computations have not been finished), but the formal analogies between the two methods leaves little doubts about its validity.

A Lie-algebra structure of mutual couplings between self-interacting vector mesons from first principles would mean a symmetry relation between coupling strengths which is not imposed but rather the result of the spacetime quantum causality properties of QFT without any support through quantization from classical gauge theory. This is very different from our usual understanding of inner symmetries according to which every symmetric theory can be converted into a less symmetric model (more coupling parameters) while maintaining its field content.

At this point it is helpful to recall the standard conceptual relation between the causal localization principles underlying QFT and internal symmetries. If one starts from a QFT in which local fields transforms under a unitary compact group which leaves the vacuum state invariant, the observable algebra associated with the field algebra is simply the fixed point algebra of the field algebra under the action of the symmetry group.

The important conceptual enrichment through the Doplicher-Haag-Roberts superselection theory was that starting from an observable algebra with a causal localization structure (a "net of spacetime localized algebras" [13]) one can reconstruct a "field algebra" (with a nontrivial compact group acting on it) from the superselection structure of its inequivalent unitary representations. Observable algebras are defined solely in terms of spacetime causality concepts of operator algebras; no other group than the spacetime Poincaré transformations play a role in their definition. Yet an observable algebra leads in a rather canonical way to a field algebra together with a compact group acting on it.

This construction is somewhat astonishing since at first glance the causal localization properties of local observables seem to have no connection with group representation theory; the net of localized observable algebras is covariant under the Poincaré group (including the TCP operation) but there is no direct indication to gauge symmetry, be it global or local. Yet the classification of unitarily inequivalent local representations of the spacetime-indexed set of local algebras (the local superselection sectors of the observable algebra) leads indeed to a field algebra and a compact groups acting on it.

There is no analog in classical physics, all the classical Lagrangians with internal symmetries are obtained by reading back QFT into the classic realm; classical Maxwell theory and Einstein's field theory of gravitation do not contain internal symmetries. The quantum concept of inner symmetry entered particle physics through Heisenberg's introduction of isospin into nuclear physics.

In a perturbative view of QFT with inner symmetries one can always think about keeping the same field content but using a less symmetric coupling (more independent coupling parameters). In fact the terminology "symmetry" is only meaningful in contrast with less or no symmetry. The apparent high symmetry of self-interacting $s = 1$ vector potentials has no analog for interactions between $s < 1$ fields.

The construction uses only the spacetime causal localization properties; the group theory is hidden in the composition structure ("fusion rules") of the localization-preserving inequivalent representations (endomorphisms) of the observable algebra [13]. In this way the origin of global symmetries (inner symmetries) is fully accounted for in terms of spacetime quantum localization concepts.

This is very different for local gauge symmetries. Using our insight into the relation between localization and Hilbert space positivity for interactions involving $s \geq 1$ interactions we may say local gauge theory results from forcing a $s = 1$ Hilbert space setting which requires to use string-local fields, into a point-local setting. The prize for this is that the pointlike fields act in a Krein space and that we can extract physical objects by noticing that the pointlike formalism comes with an unphysical but nevertheless useful symmetry which acts in Krein space and whose fixed point algebra is the physical algebra of *local observables* which is generated by point-local physical fields. Although the SLFT Hilbert space setting has no gauge symmetry, it contains a Lie-algebraic structure in its f_{abc} couplings between string-local fields.

This explains why theoreticians using methods of algebraic QFT had [13] problems with gauge symmetries and why a foundational understanding requires new ideas [23]. The hope is that perturbative observation in a Hilbert space setting can be combined with nonperturbative algebraic results from LQP in order to obtain further clarification. It would be very interesting to see whether the perturbative results in this paper can be backed up by structural theorems about possible connection between interacting QFTs involving $s = 1$ fields and the need to go beyond the point-local setting of Wightman fields.

6 An Outlook

As stated in the introduction, the principle motivation for writing this paper is to direct attention to the beginnings of a new development in QFT whose aim is to preserve renormalizability within a Hilbert space setting for interactions involving $s = 1$ and possibly higher spin fields. In the present attempt we omitted mathematical details and focused on those conceptual properties which distinguish the new setting from that of gauge theory.

The clash between zero mass point-local vector potentials and Hilbert space positivity, which is resolved by the use of covariant string-local potentials, suggests to use such potentials also in the massive case. In this way one constructs string-local siblings of the point-local Proca potentials which pass smoothly to their massless counterpart. The important gain of such a construction is that the short distance behavior is lowered from $d_{sd}^P = 2$ to $d_{sd} = 1,$ a reduction which gauge theory achieves at the prize of abandoning Hilbert space positivity and working instead in a Krein space.

The use of this observation leads to an extension of renormalization theory to interacting string-local vector mesons. Since $d_{sd} = 1$ for all sl $s \geq 1$, the mere fulfillment of the $d_{sd}^{int} \leq 4$ power counting restriction for controlling short distances is easy, but the danger is now that the interaction leads to *completely delocalized* fields in higher orders. In order to avoid this the first order string-local interaction density L must be part of a so-called L, V_μ pair which turns out to be a strong restriction. Its fulfillment in every order leads to normalization conditions which must be fulfilled in all orders of $s = 1$ interactions. This has a formal similarity with the fulfillment of BRST invariance $\mathfrak{s}S = 0$ in every order of perturbation theory.

The new setting does not invalidate gauge theory but it highlights its restricted physical range. GT permits a correct description of the S-matrix of interaction massive vector mesons and local observables, but the physics behind gauge-variant fields remain outside its physical range. This includes the problem of quark confinement and also the spacetime understanding of scattering problems in QED. Within the confines of gauge invariance, gauge theory is a very successful placeholder of a QFT of $s = 1$ particles.

Disregarding structural theorems (TCP, Spin&Statistics, derivation of large time scattering,...) whose proof requires Hilbert space positivity, the BRST perturbative formulation accounts for the perturbative gauge-invariant local observables (field strength, currents) and, what guarantied the success of GT for the Standard Model, the perturbative unitary S-matrix. What is missing are the matter fields which relate the world of the causal localization principles of QFT with the measurable world of Wigner particles. Physical consequences of long distance behavior (infrared problems) are outside the physical range of GT and must be investigated in the Hilbert space setting of sl fields. Whereas one expects that the asymptotic freedom property of QCD in the gauge setting will be confirmed in the sl setting, the long distance behavior remains outside GT's physical range.

In order to go beyond gauge theory the SLFT Hilbert space setting for the S-matrix must be extended to the calculation of string-local correlation functions, a task which goes significantly beyond the construction of the string-independent S-matrix. Here consistency demands that the counterpart of the on-shell string independence should be the independence of vacuum expectation values of string-local fields on $e's$ of *inner* propagator lines. The perturbative formulation of this idea is presently being studied.

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