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To the Memory of Alexander Grothendieck: a Great and Mysterious Genius of Mathematics

Em memória de Alexander Grothendieck: um grande e misterioso gênio da matemática

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Abstract: In November 2014 Alexander Grothendieck passed away at the age of 86. There is no doubt that he was one of the greatest and most innovative mathematicians of the 20th century. After a bitter childhood, his meteoric ascent started in the Cartan Seminar in Paris, it led to a breakthrough while he worked in São Paulo, and to the Fields Medal. He introduced numerous new concepts and techniques, which were involved in the groundbreaking solutions to long-standing problems. However, dramatic changes were still ahead of him. In recent years hardly anybody knew where he was living, and even if he was still alive; he had withdrawn to a modest life in isolation. Also beyond his achievements in mathematics, Grothendieck was an extraordinary person. This is a tribute of his fascinating life.

Keywords: Modern History of Mathematics; Points in Geometry and Physics; Science, Environment and Peace; Activism and Meditation.

Resumo: Em Novembro de 2014 Alexander Grothendieck faleceu aos seus 86 anos de idade. Não há duvidas de que ele foi um dos maiores e mais criativos matemáticos do século XX. Após uma infância amarga, sua ascensão meteórica iniciou no Seminário Cartan em Paris, o que o levou a um avanço enquanto ele trabalhava em São Paulo, e a Medalha Fields. Ele introduziu numerosos novos conceitos e técnicas, que foram envolvidos nas soluções inovadoras de problemas de longa data. No entanto, mudanças dramáticas ainda estavam por vir. Nos últimos anos, quase ninguém sabia onde ele estava morando, ou até mesmo se ele estava vivo; ele havia se retirado para uma vida modesta em isolamento. Também para além dos seus méritos em matemática, Grothendieck foi uma pessoa extraordinária. Isso é um tributo a sua fascinante vida.

Palavras chave: História Moderna da Matemática; Pontos em Geometria e na Física; Ciência, Meio Ambiente e Paz; Ativismo e Meditação.

1. AN UNUSUAL FAMILY

Alexander Grothendieck's life was dominated by turbulence and radical tuning points. As a constant feature, however, he followed consistently his own path, keeping away from anything established.

To capture the spirit of his highly unusual biography,[1] we have to start with his parents. His father —

[1] We adopt the biographical information essentially from Refs. [1, 2], but

whom Grothendieck honored very much — was Alexander Schapiro (1890-1942), born in the Russian town Novozybkov (in the border region with Belarus and Ukraine), in an orthodox Jewish community. Still very young he joined an armed anarchist group, which was captured in 1905, after the failed attempt to overthrow the tsarist regime. All members were executed, except for Alexander, who was pardoned to life in prison due to his youth. About ten years later he escaped, and readily joined another anarchist army, this time in the Ukraine. He was captured again, sentenced to death, but he managed to escape once more (though he lost his left arm).

Then he lived under the name Alexander Taranow in Berlin (and other cities), where he worked as an independent photographer. Around 1924 he met Hanka (actually Johanna) Grothendieck (1900-57), told her husband: "I will steal your wife" — and he proceeded in doing so. Hanka was a far-left activist too, and she tried to become a journalist and writer, but — despite her talent — she could not publish much. In 1928 their son Alexander Grothendieck was born; Schurik — this was his nickname — lived for the first five years with his parents and a half-sister in Berlin (in the Scheunenviertel).

2. YOUTH DURING WORLD WAR II

In 1933, when the Nazis came to power, the situation was getting too dangerous for Schurik's father, who flew to Paris. Hanka joined him soon, and left Schurik with a foster family in Hamburg. There he attended school from 1934-39 and lived in the home of Wilhelm Heydorn — a former military officer and priest, who turned pacifist and atheist. Alexander's Jewish ancestry was kept secret, but in 1939 Germany was getting too dangerous for him as well, in particular because his foster parents opposed the Nazi regime. He was put on a train to France where he met his parents again; they were back from the Spanish civil war, having supported an anarchist group. In 1940 the family was imprisoned in internment camps by the Vichy regime, which collaborated with the Germans. Two years later, Alexander Taranow was extradited to the Nazis, he was deported to Auschwitz where he died.

Alexander (Schurik) Grothendieck a the age of 12.

the most complete source is Ref. [3]. We apologize for not quoting them each time.

Also in 1942, 14-year-old Alexander arrived in Le Chambon-sur-Lignon, a small town in the Massif Central, which was a center of resistance against the German occupation. Here Alexander attended a school, which was devoted to the spirit of pacifism. When there were raids by the Gestapo, he and other pupils hided in the forest for a couple of days, divided into small groups [4]. In 1945 he finished his *baccalaureate.*

After the war his mother Hanka was released, and Alexander was closely attached to her until her death in 1957.[2] They moved to Montpellier, where Alexander studied mathematics, and received a modest scholarship. The local university was not very helpful to him, so he resorted mostly to autodidactic studies. He was particularly interested in a deep understanding of space and geometry, starting with the notion of a *point,*[3] and he elaborated by himself a generalized concept of integration.[4]

3. A FAIRYTALE-LIKE CAREER

In 1948 Alexander was awarded a fellowship to go to Paris, where he got in contact with mathematical research; in particular, he attended the famous *Cartan Seminar.* He was not shy to discuss with famous scholars, he was ambitious and passionate; later he wrote "j'étais un mathématicien: quelqu'un qui *fait* des maths, au plein sens du terme – comme on *fait* l'amour" [4]. Initially he hoped for his independent work to provide a quick Ph.D., but he was told that he had essentially re-discovered the Lebesgue integral (which had been known since the early 20th century). Also later, as a highly established mathematician, he always followed his own ideas, rather than studying the literature (he got informed about relevant results in discussions).

Since Alexander wanted to explore Topological Vector Spaces, Henri Cartan and André Weil recommended him to move to the University of Nancy, in Northern France, where two leading experts were working: Jean Dieudonné and Laurent Schwartz — the latter was also a pioneer in Distribution Theory, and he just won a Fields Medal.[5] He showed his new student his latest paper; it ended with a list of 14 open questions, relevant for locally convex spaces. Alexander went ahead and introduced new methods, which allowed him to

[2] Hanka died of tuberculosis, probably as a consequence of her confinement during the war. However, she still witnessed her son's upcoming world fame.

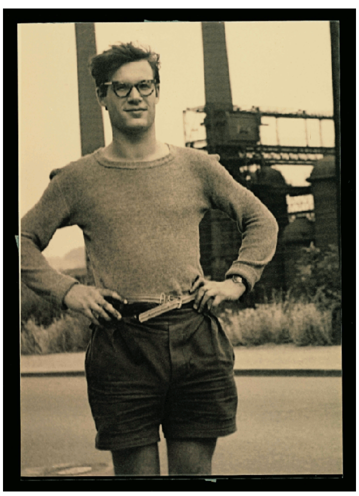
[3] Already in High School it upset him that his text books never gave a satisfactory definition of terms like *length, area* and *volume.*

[4] Apparently Hanka obstructed his contact with girls, which was certainly favorable for the intensity of his studies. Nevertheless, in his later life he had five children with three women. One of them, Mireille Dufour, was his wife in the 1960s. She was from the Normandy, a little older than Alexander, and she also had links to the anarchist movement in Spain. It is reported that even their marriage had an touch of anarchy; now Alexander had a variety of affairs.

[5] The Fields Medal is the highest distinction in mathematics, sometimes regarded equivalent to a Nobel Prize (though there is an age limit of 40). It is awarded at the *International Congress of Mathematicians* (ICM), the largest mathematical conference, which is held once every four years (it coincides with the year of the Soccer World Cup).

solve *all* these problems within a few months! A mathematical superstar appeared, at the age of 22, with a chaotic youth and sparse education.

Despite his success, it was difficult for him to find a job in France, in particular because he was stateless. His advisors mentioned the unfortunate situation of this young genius, and found a visitor position for him at the University of São Paulo, where Alexander stayed from 1952-54 [5]. The contact was established by Paulo Ribenboim, a Brazilian student of the same age as Alexander, who also worked in Nancy (later he became a prominent mathematician in Canada). In that period, legendary president Getúlio Vargas was in power in Brazil, and Alexander finished his Ph.D. thesis on *Tensor Products and Nuclear Spaces* (the second term he had introduced himself). According to Dieudonné, at that time he had already results that would have been sufficient for six theses, covering also functional analysis. He published in Brazilian journals (in French), where he introduced the Grothendieck Inequality, and he lectured on Topological Vector Spaces; his lecture notes were published as well [6]. Meanwhile he started to shift his focus of interest towards *Algebraic Geometry* — the field where he ultimately had his strongest impact; it involves the systematic analysis of the geometric properties of the solutions to polynomial equations.



The young mathematician Grothendieck, in a picture taken by Paulo Ribenboim in 1951.

Alexander worked with full intensity, essentially he only paused to sleep and eat. Schwartz, his Ph.D. advisor, asked Ribenboim to encouraged him to do occasionally other things in life — at that time without success. His mother visited him in Brazil, which presumably again prevented him from being side-tracked (cf. footnote 4). His colleague Chaim Hönig (who came to Brazil before the war, as a German refugee) later remembered that Alexander led a "spartan and lonely existence", living sometimes of milk and bananas, and he got frustrated when he failed to solve a problem despite hard work. Still, the problems that he did solve, and the methods that he introduced, boosted his meteoric career.

4. THE GOLDEN ERA AT IHÉS, 1958-70

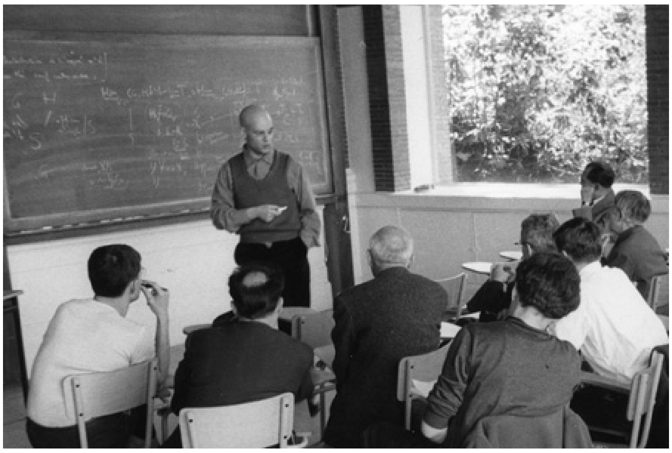
After a short stay in Kansas, Grothendieck returned to France. Together with Dieudonné and Jean-Pierre Serre, he soon worked at the newly founded *Institut des Hautes Études Scientifiques* (IHÉS) near Paris, which became famous for its research in mathematics and theoretical physics. Grothendieck led a group of brilliant young mathematicians. This era of excellence, 1958-70, coincided with the climax of the *Bourbaki* group, which Grothendieck was in contact with (for some years he was a member). When a visitor noticed that the library of the new institute was rather incomplete, Grothendieck replied: "We don't read books, we write them" [1].

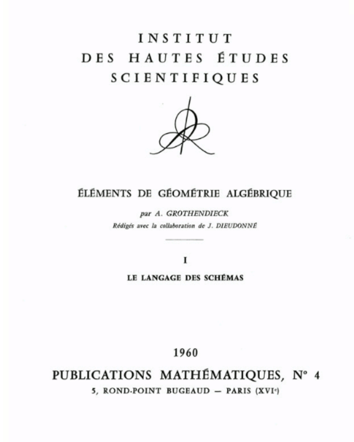
His former colleague Pierre Cartier asserts that he run "one of the most prestigious mathematics seminars that the world has ever seen" [7]. It attracted top mathematicians from France and all over the world. Session could take 10 to 12 hours, leading to improvised notes that Grothendieck gave to Dieudonné, who would then rewrite them in a neat form. Grothendieck is remembered as an excellent teacher, who explained also "trivial" points patiently, with a talent to suggest the appropriate subject to each member of his group. His motivation was simply to *understand,* not competition.

He had anticipated his research program for these years in a plenary talk at the *International Congress of Mathematicians* in Edinburgh, 1958. His style was to search for ever increasing *generality and abstraction* (which was a trend of mathematics in the 20th century), introducing accurate new terms and concepts, and working out their properties. His colleague John Tate emphasizes that Grothendieck found again and again exactly the right level of abstraction, so he was neither dealing with a special case, nor with a pointless "vacuum". This led to thousands of pages on the merger of Algebraic Geometry, Arithmetics and Topology. His interest was mostly in new, generic concepts, like *schemes, étales, toposes* and *motives*; for popular descriptions we refer to Refs. [7, 8], or (more detailed and technical) Ref. [9]. Grothendieck hardly appreciated applications in natural science, like physics.[6] Even the proofs of explicit mathematical theorems were an inspiration for him, but not really the ultimate goal. However, Gerd Faltings' proofs of the Tate and the Mordell Conjecture, as well as Andrew Wiles' proof of Fermat's Last Theorem, can all be viewed as applications of *motives.*

For a number of years, Grothendieck proved step by step aspects of the Weil Conjectures (dating back to 1949), which inspired amazing new concepts. Later, in 1974, his former student Pierre Deligne proved the last point of these conjectures. However, he invoked a classical result, deviating from the program of a generalized context, which employs *motives,* as sketched in the IHÉS seminars. His mentor appreciated this success, but he was still somehow disappointed.

[6] Occasionally he got a bit interested in biology, encouraged by a friend in Romania.





INSTITUT
DES HAUTES ÉTUDES
SCIENTIFIQUES

ÉLÉMENTS DE GÉOMÉTRIE ALGÉBRIQUE

par A. GROTHENDIECK
Rédigés avec la collaboration de J. DIEUDONNÉ

I

LE LANGAGE DES SCHÉMAS

1960

PUBLICATIONS MATHÉMATIQUES, N° 4
5, ROND-POINT BUGEAUD — PARIS (XVI°)

On the left: A seminar at the Institut des Hautes Études Scientifiques (IHÉS) near Paris.
On the right: front page of Grothendieck's book "Éléments de Géométry Algébrique".

In 1970, at the age of 42, Grothendieck abruptly resigned from IHÉS, and entered a completely different phase of his life. Soon afterwards his brilliant research group fell apart.

5. NEW INTERESTS, AND A NEW LIFE STYLE

Until 1970 Grothendieck's life was almost non-stop focused on mathematics (later he called it his "long period of mathematical frenzy" [4]), and his life style seemed rather bourgeois. People described him as friendly, direct, by no means arrogant, idealistic but — for issues beyond mathematics — somewhat naïve. However, other issues of the world did come to his mind, and gradually became dominant. In particular, he felt strongly committed to *pacifism.*

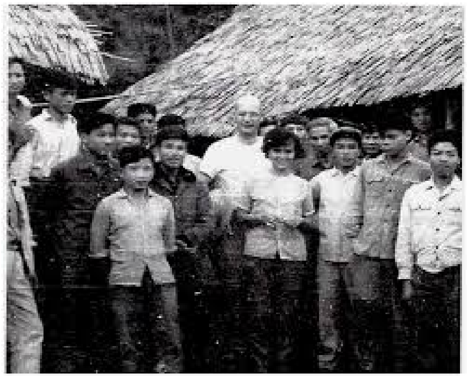
Since the late 1950s he was wearing Russian peasant cloths and shaved his head, in the memory of his father, and he liked to wear sandals made of tire. When he was invited to Harvard University in 1958, he criticized the visa requirement of swearing that he would refrain from subversive actions. Like other prominent mathematicians, he opposed to the French colonial war in Algeria, 1954-62. In 1966 he was awarded the *Fields Medal,* which he was supposed to receive in Moscow, but he did not show up, referring to two Russian writers who were arrested.[7] Still, he did visit Eastern European countries at other occasions, and his ideas had a remarkable influence among Russian mathematicians, like Vladimir Drinfeld, Maxim Kontsevich, Yuri Manin and Vladimir Voevodsky.

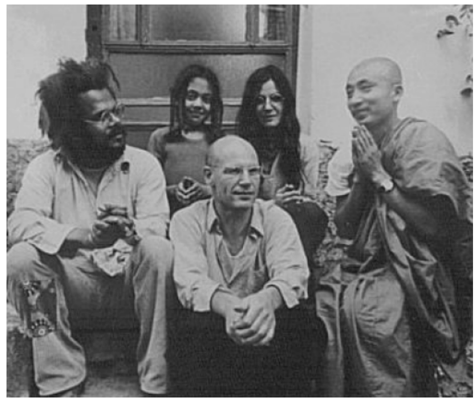
Meanwhile the students movement of the 1960s gained more and more momentum, and culminated in May 1968 in Paris. Grothendieck was strongly impressed, but he found himself on the wrong side — he felt attracted to the rôle of an outlaw, not to the establishment. He sympathized with the movement, which involved in part anarchist ideas, but he did not attend public rallies; also in this respect he followed his own path.

In 1967 Grothendieck received a request from Hanoi, asking for literature about Algebra and Algebraic Geometry. He had not been aware that there was mathematical research going on in North Vietnam, even during the worst period of the second Vietnam War, and he provided as much material as he could. Moreover, Grothendieck decided to travel to North Vietnam himself to give lectures. After a first part in Hanoi, the cluster bombing by the US Air Force intensified so much[8] that he and his Vietnamese attendees (among them Ta Quang Buu, mathematician and Minister of Higher Education and Technology, who frequently asked questions) moved to a hidden place in the forest to continue the lecture (during the breaks, Grothendieck went to a near-by river to wash his cloths). In 2013 Neal Koblitz, a mathematician from the USA, visited this place, and he was intrigued by the fact that the course given there could have been presented as well at Harvard University, where Koblitz had been studying at that time [10]. After his return to France, Grothendieck gave talks about his visit and wrote a detailed report [11], which informed the world about the mathematical community in North Vietnam. While he described the state as somewhat over-regulated, his report firmly expresses his sympathy for the underdogs of this destructive war, which lasted for 30 years in total, and left nearly 4 millions of people killed.

[7] IHÉS director Léon Motchane received the Fields Medal at the *International Congress of Mathematicians* in Moscow, on Grothendieck's behalf.

[8] A week after Grothendieck's arrival, the campus of the Hanoi Polytechnical Institute was hit by delay-action bombs, which killed two mathematicians.





On the left: Grothendieck in the Vietnamese rain forest, 100 km from Hanoi.
On the right: a picture of 1975, characteristic for his new life style.

When Grothendieck quit IHÉS in 1970, the reason he gave was that he had discovered that his institute received funds for the French military. Actually this was known before, and many scientists objected, so in 1969 an agreement was reached to stop this practice. However, only one year later the agreement was broken. Grothendieck tried to convince his colleagues to resign in protest, but it was only him who really did so. Although this was about a minor fraction of the IHÉS budget, it was an ethical problem for him — for instance, his Ph.D. advisor Schwartz had been working hard to transform the École Polytechnique from a military to a civilian orientation. Research with military purposes was not acceptable for Grothendieck, and this was also a reason why physics was suspicious to him, keeping in mind Hiroshima and Nagasaki.[9]

However, there might have been other reasons involved in this radical turning point of his life: conflicts with colleagues at IHÉS, a decrease in creativity, and the consciousness that his ambitious goals would never be completed.[10] Was there also some burnout or mid-life crises involved?

In any case, Grothendieck changed his life style, he got separated from his wife Mireille, and opened *communes*, first in Paris and later in Southern France. There he lived with a variety of people, at times three of his children were among them, and political meetings were held. Meanwhile he lectured on a temporary basis, first at the *University Paris-Sud* in Orsay, and then in the *College de France.* In his courses he took the opportunity to discuss also issues like the threat by nuclear weapons. This attracted a broad audience, but the *College* direction was not amused, and denied him a permanent position, even though he was one of the most famous mathematicians in the world.

In 1973 he moved back to the University of Montpellier (although its Mathematics Department did still not match his standard) and gave lectures on all levels. He was friendly to his students, who dubbed him *Alexandre le Grand,* he distributed organic apples, and gave inspiring courses. He did not run a highly ambitious seminar anymore, but he still had several Ph.D. students (and he got angry when the *Springer* Publishing House declined publishing a thesis). Still he led excellent research, but the French research agency CNRS only provided marginal support.

From 1973-79 he lived in the tiny village Olmet-et-Villecun, 50 km from Montpellier, in a simple house without electricity (he used kerosene lamps to work at night). He did not hesitate to give shelter to homeless people. Generally, his home was open for everybody, and it became a meeting place for all kind of people, including the *hippie movement.* In 1977 it was raided by the police, which were looking for anything possibly illegal. All they found was a Japanese citizen, who was staying there, and whose French visa had expired. He was an peaceful person, who had studied mathematics, but at that time he was a Buddhist monk. Half a year later (when the monk had long left France), Grothendieck was actually accused for giving shelter and food to a foreigner "in an irregular situation". He defended himself with a passionated speech, and many mathematicians gave him public support, but he got convicted to a heavy fine and a six-months suspended sentence.

6. ENVIRONMENTAL AND PEACE MOVEMENT

Meanwhile Grothendieck questioned intensively the sense of scientific research — he reported that in many discussions, nobody could really give a reason for it. He got more and more concerned about ecological problems and militarism, in particular the danger of a nuclear war. He was convinced that everyone, who was given the relevant infor-

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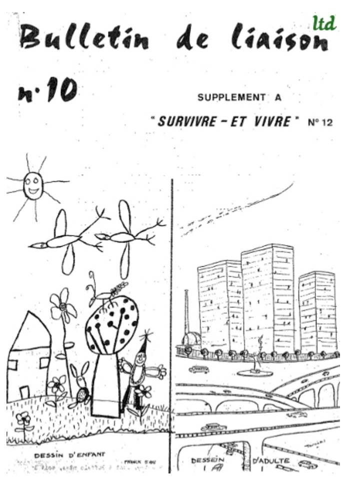
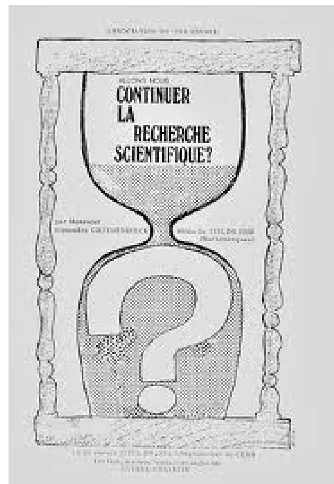
9 On the other hand, although his mathematical style was very abstract, some of his concepts did propagate into theoretical physics, in particular in constructive field theory, which was elaborated in the 1970s. A later example is *non-commutative geometry,* which was — on the formal side — strongly developed by Alain Connes, who adopted ideas by Grothendieck. It became a wide-spread fashion in theoretical physics in the late 1990s. Yet another example is the *Atiyah-Singer Index Theorem,* which relates the zero modes of a chiral Dirac operator to the topological charge of its gauge background. Its derivation involved Grothendieck Groups, which emerged from his new proof of the Riemann-Roch Theorem, and which became later a point of departure for K-Theory.

10 He had outlined a monumental program to write a series entitled *Éléments de Géométry Algébrique* in 13 volumes. "Only" four volumes appeared, comprising about 1800 pages.

mation, would follow his logical arguments and agree with his conclusions, and that he had a mission to spread this message.

When he was invited to lecture at a Summer School in Montreal, he accepted under the condition that he could not only lecture about mathematics, but also about the threats to humanity. In fact, some young mathematicians followed his ideas, and became activists as well. He also gave double lectures in the USA, where he further supported the rights of Native Americans. A Ph.D. student named Justine Skalba was particularly excited about his charisma and intelligence. She followed him to France as his partner for a few years (their son was born after the relation had ended, but he later did a Ph.D. in mathematics). Justine remembers a rally in Avignon that they attended together; when it was harassed by the police, Grothendieck knocked down two policemen, and got arrested.





On the left: a page of the magazine "Survivre et Vivre": it compares the drawing by a child and by an adult. On the right: announcement of Grothendieck's talk at CERN, 1972, entitled "Are we going to continue the scientific research?".

Together with another two prominent French mathematicians, Claude Chevalley and Pierre Samuel, he founded a group called *International Movement for the Survival of the Human Race.* It published the magazine *Vivre,* or later *Survivre et Vivre,* with emphatic calls for peace and against pollution, discussions of the impact of science, and a critical view of the consumerism-minded society (in part inspired by the philosopher Herbert Marcuse). He wrote a considerable number of articles for this magazine, which appeared from 1970-75; copies are available in the internet [12].[11]

When Grothendieck attended the *International Congress of Mathematicians* in Nice, 1970, he installed a desk to distribute this magazine, together with his eldest son Serge (from a premarital relation), and tried to recruit new members to his movement (with limited success). Dieudonné, who was responsible for the event, stubbornly objected, until they moved the desk outside the building, but there they got in trouble with the police.

At a Summer School 1972 in Antwerp, Serre gave the opening speech. Grothendieck vociferously interrupted his former IHÉS colleague, to speak out against NATO, which had sponsored this event.[12] He did not hesitate to be provocative *(l'enfant terrible)*, even if this led to resentment with long-term friends and collaborators.

Of course he was confronted with the reproach of overdoing it in an immature manner. However, for instance a talk of two hours, including an extensive discussion with the audience, that he gave at CERN in 1972 (now accessible on YouTube) sounds calm and thoughtful: he was aware, of course, that CERN does not focus on nuclear research (in contrast to its name). He explained why he took distance from the scientific community, with its competition and pressure to publish, which are unjust and unfavorable for creativity, and which keeps researchers working without ever wondering for what reason. He also recalled mathematicians who had committed suicide. He further pointed out why he now considered actions against the threats to humanity — like nuclear weapons — far more important.

Despite his conviction and arguments, the use of his academic reputation and his rhetorical skills, his group remained small. It mostly attracted people who had similar views before, and around 1973 a trend of dissolution set in. Grothendieck was disappointed and considered his efforts as a failure. He concluded that people, even scientists, were blind to the dangers to the world, and do not behave rationally.

[11] It goes without saying that this movement was also confronted with dismissive reactions; *e.g.* Ref. [7] describes it as a "dooms-day sect", which was "obsessed by pollution".

[12] As a reaction, a NATO representative, who had intended to join the Summer School for a public debate, backed off. Subsequently Grothendieck was blamed for having done some kind of "damage".

Other members of this movement, like Samuel (who was an editor of *Survivre et Vivre* until 1973), patiently carried on efforts along these lines. From today's perspective, these actions appear as pioneering work for the peace and environmental movement, which later became influential — to some extent — in Europe and beyond. Also within science, ecological concerns were later acknowledged, *e.g.* with the 1995 Chemistry Nobel Prize for demonstrating the danger to the Earth's ozone layer. Today, for instance global warming due to human activities is only disbelieved by some people who are at odds with science. So we could ask if Grothendieck's activism in the early 1970s was really immature, or if he was rather ahead of his time?

7. SPIRITUALISM AND ISOLATION

Frustrated about his modest success as an activist, Grothendieck slowed down his appeals to the public. He kept writing long manuscripts, with magnitudes of 1000 pages, like *La Longue Marche à travers la théorie de Galois,*[13] *A la Pursuite de Champs* and *Esquisse d'un Programme* with ideas for future mathematics. Indeed, that *Programme* was worked out to a large extent by the young mathematicians Leila Schneps and Pierre Lochak, who were impressed by its farsighted vision. They contacted Grothendieck, who suddenly expressed his interest in *physics* and asked for literature about it [1] (although he regretted its lack of rigor). Later they also initialized the *Grothendieck Circle,* which created an informative web page [12], and Schneps edited an overview over Grothendieck's mathematical achievements [13].

In the period 1983-88, Grothendieck wrote a stylistically brilliant book entitled *Récoltes et Semailles* [4], where he reviews his life and work, supplemented by all kinds of elements, like love poems (in German) and (sometimes critical) comments on the mathematical community and former colleagues. In Section 2.20 he addressed modern physics. From a mathematical perspective, he did not consider Einstein's Theory of Relativity very interesting, although he appreciated its importance for our paradigm of space-time. Mathematically, however, he described the transition from Newton's Theory to Relativity like a change from one French dialect to another, whereas Quantum Theory is like a transition to Chinese. This he did find interesting, regarding his deep understanding of a *point,* and he mentions an intuitive similarity to his concept of *toposes.* We add a comment in Appendix A.

In 1988 he was supposed to receive the prestigious *Crafoord Prize* by the Royal Swedish Academy of Sciences, together with his former student Deligne, but Grothendieck declined. In a polite letter [14] he explained his reasons: first he did not need money, and about the importance of his work, time and offspring would decide, not honors. He adds that such prizes are constantly given to the wrong people, who do not need further wealth nor glorification. He asks whether this "superabundance for some" is not provided "at the cost of the needs of others"? Finally, he points out that agreeing to "participate in the game of *prizes*" would imply his "approval to the spirit ... of the scientific world", where ethics has "declined to the point that outright theft among colleagues (especially at the expenses of those who are in no position to defend themselves) has nearly become a general rule".

Also in 1988 he retired from Montpellier University, and in 1991 even from *society;* he broke off contacts with almost everybody, including his family. He withdrew to a modest life in a hamlet in the French Pyrenees, not far from Vernet Camp (the redoubtable camp, where his father had been imprisoned before being deported to Auschwitz). He still wrote the mathematical program *Les Dérivateurs* (about 2000 pages), which he handed over to a friend. On the other hand, he once burned a huge amount of notes, letters and other documents, one estimates 25 000 pages. His main interest now shifted to spiritualism and meditation, and he entered the final, Steppenwolf-like phase of his life.

For quite some time, since the 1970s, he was strongly interested in Buddhism. There are hints that this helped him to relax from the pressure of productivity, and to improve the relation with his ex-wife Mireille. He was a strict vegetarian and received Buddhist teachers. He was also fascinated by the symbols of Yin and Yang [4], and characterized his style of research as Yin.[14] Later, however, he moved on to a mystic and unconventional form of Christianity. He spent a period of starving, which endangered his health. He got much interested in dreams, which he considered the messengers of spiritual wisdom, and he studied Freud's interpretation.

As his main activity, he kept on writing; daily he spent many hours typing about his mystic experiences and ideas, which led again to thousands of pages. Although he assumed his visions to be relevant for the future society, he did not want to publish these notes. In 2010 a bizarre (but well formulated) hand-written letter appeared, where he even requested the removal of all his works from the libraries.

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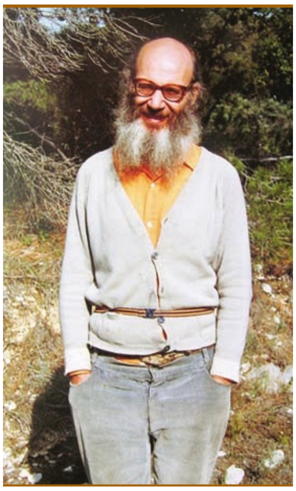
[13] Grothendieck had a special esteem for Évariste Galois, whom he called his *frère de tempérament* [4]. In fact the two mathematical geniuses had a number of points in common: a push for a new level of abstraction and a major interest in the *relation* among mathematical objects; now the term "Grothendieck's Galois Theory" is used. Moreover, both had an early end of their career (Galois' case was far more extreme), and they were radical activists for what each one considered, in his epoch, as an urgent progress of society.

[14] Deligne describes a proof by Grothendieck as a lengthy sequence of trivial steps, "nothing seems to happen, but yet at the end a highly non-trivial theorem is there". This is in contrast to Serre's Yang-style of striving for a solution in one strike [9].



Grothendieck in his older days.

Only very few people knew where he lived and promised not to spread this information. He did not have a postal address nor a telephone, let alone internet, and he did not receive uninvited visitors. At last the world learned that he has died, on November 13, 2014, in the village of Saint-Lizier in the French Pyrenees — Alexander Schurik Grothendieck, rest in peace.

Apêndice A: POINTS IN PHYSICS

When Grothendieck writes that he finds Einstein's Relativity mathematically "banal" [4], he includes General Relativity. That is based on mathematics of the 19th century, in particular Differential Geometry, which is actually non-trivial (by common perception). Nevertheless, his statement becomes plausible if it addresses a specific, fundamental understanding of geometry, in particular the very nature of a *point;* we have mentioned before that this issue has haunted Grothendieck since his youth. In this regard, he considers Quantum Mechanics far more interesting.

In Ref. [4], p. 69, he writes: "And these *probability clouds,* which replace the certain material particles that we had before, remind me strangely of the elusive *open neighborhoods* which inhabit the toposes, such evanescent ghosts, which surround fictitious *points,* which keep on attaching themselves, in contrast to a recalcitrant imagination".

The state of a quantum mechanical particle is given by a (time-dependent) vector in a Hilbert space, in Paul Dirac's notation $|\psi(t)\rangle$, and the position eigenstates $|x\rangle$ form a basis. The scalar product $\psi(t,x) = \langle x|\psi(t)\rangle$ is the particle's wave function, and $|\psi(t,x)|^2$ its "probability cloud".

Hence standard Quantum Mechanics is still formulated in a standard coordinate space with a continuum of sharp points, where the particle wave functions are accommodated. The spatial resolution is not limited in principle, if sufficiently large momenta are available to resolve it. So a possible analogy to *toposes* could rather refer to *phase space,* where points do have a conceptually limited resolution — given by Heisenberg's Uncertainty Relation — hence there are only *fuzzy* points.

On the other hand, angular momenta only take sharp, discrete values. A deep understanding of this interplay between discrete and continuous, sharp and fuzzy, could have been something for Grothendieck's taste.

There have been attempts to employ *toposes* in an unconventional formulation of Quantum Mechanics, with the hope to alleviate problems with the interpretation of measurements on quantum systems; for reviews see Refs. [15].

We still do not have a convincing merger of Quantum Physics with General Relativity, but a Lorentz covariant formulation (hence a conciliation with Special Relativity) is accomplished by Quantum Field Theory. But again, the usual formulation, which encompasses in particular the (tremendously successful) Standard Model of Particle Physics, employs a simple Minkowski space. Thus the fields are still functions of sharp points in space-time — this does still not seem exciting in view of Grothendieck's particular motivation, at least at first sight.

One might object that some regularization of high momentum contributions is required, to suppress the omnipresent *ultraviolet singularities.* This corresponds to some kind of truncation of short distances, *i.e.* a somehow granular space-time (its structure depends on the regularization scheme).[15] In theories like Quantum Chromodynamics (the sector of the Standard Model that describes the strong interaction), this truncation can be fully removed at the end of the calculation, *i.e.* one extrapolates to the *continuum limit,* hence it is just a mathematical trick. Also the electroweak sector was shown to be *renormalizable,* which allows to remove the cutoff at the end. On the other hand, in the Higgs sector of the Standard Model, a complete removal of the truncation would also remove the interactions, so the Higgs field becomes free and does no longer do its job of providing the elementary particle masses. Usually physicists don't worry much about this property (which is known as *triviality*), since a huge momentum cutoff, extremely far above the experimentally accessible regime, is sufficient to justify the observed mass of the Higgs boson.[16] However, from a fundamental geometric point of view, one might pay more attention to this aspect.

Based on reports of people who talked to Grothendieck in his old days, he seemed interested in the question if the constants of Nature are related by *rational* ratios. Usually we do not assume that (*e.g.* we do not have reasons to expect the fine structure constant $\alpha = e^2/(2\pi\varepsilon_0\hbar c) \simeq 1/(137.036)$, or the ratios of particle masses, to be rational), but it is certainly the case for the electric particle charges. Dirac gave an explanation for this property, but it requires the existence of at least one magnetic monopole, for which we do not have any evidence. In the Standard Model and some (though not all) of its variants — in particular incorporating neutrino masses — that property can also be deduced from the theoretical requirement of gauge anomaly cancellations.

In the framework of the diverse attempts to achieve compatibility of Quantum Theory also with General Relativity,

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[15] The scheme that works at finite interaction (*i.e.* beyond perturbation theory) performs indeed a reduction to a "lattice" of discrete (but sharp) space-time points.

[16] It is now getting popular to interpret the Standard Model as a low energy effective theory, valid up to some energy range above the scale of experiments, which is sufficient for practical purposes.

i.e. to include also gravity, quite general arguments suggest an extension to a *pure space-time uncertainty relation,* which should be manifest at extremely short distances, of the order of the Planck scale ($\approx 10^{-35}$ m), see *e.g.* Ref. [17]. That corresponds to a *non-commutative geometry,* where the coordinates in independent directions are given by Hermitian operators, which do not commute. We speculate that this could have attracted Grothendieck's interest.

We have mentioned in footnote 9 that the formal aspects of physics in a non-commutative space have been elaborated rigorously by Connes. He did apply some concepts by Grothendieck, but regarding *toposes* his comprehensive book on this subject, Ref. [16], only contains the remark: "One could base this extension of topology on the notion of toposes due to Grothendieck. Our aim, however, is to establish contact with the powerful tools of functional analysis such as positivity and Hilbert space techniques, and with K-theory."

From the physical perspective, we add that Quantum Field Theory — formulated in a non-commutative space — is plagued by severe obstacles: first we cannot instal fields for the non-Abelian gauge groups $SU(2)$ and $SU(3)$, which belong to the Standard Model. Next a non-commutative space-time entails *non-local interactions* — which also occur in String Theory — and which raise questions regarding the principle of causality. That might be acceptable if these non-local effects were restricted to tiny ranges (like the Planck scale), but for interacting quantum fields, a non-commutative space-time further gives rise to a new type of singularity in the *infrared regime.* Hence quantum effects are expected also at very *long* distances, even if one modifies the geometry only within a tiny range. This theoretical phenomenon — known as "ultraviolet-infrared mixing" — has prevented a valid confrontation with particle phenomenology, which could confirm or constrain a space-time non-commutativity in Nature, and thus the existence of fuzzy points, which bear a resemblance to the open neighborhoods in the Grothendieck toposes.

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